Field Theory and EW Standard Model

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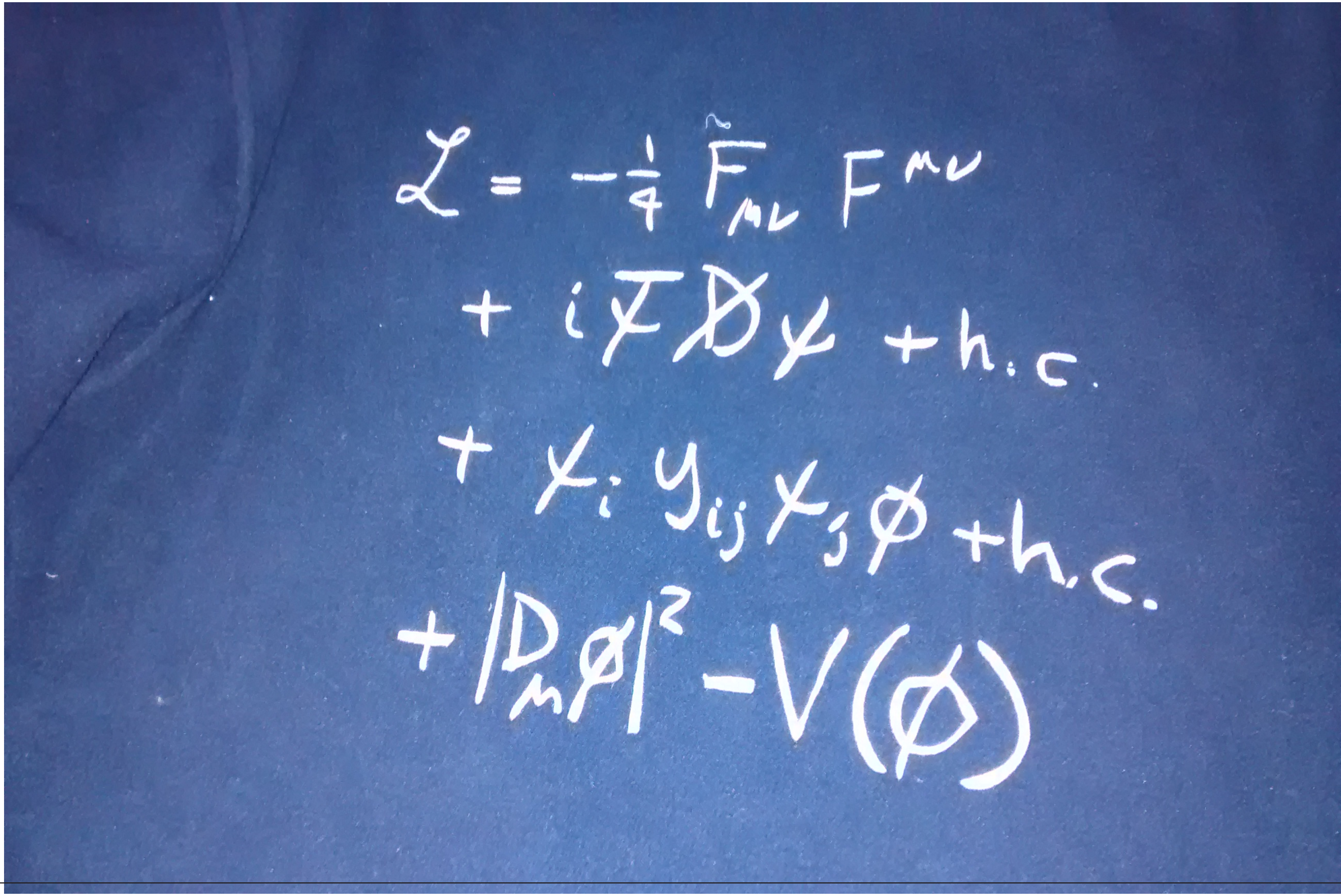
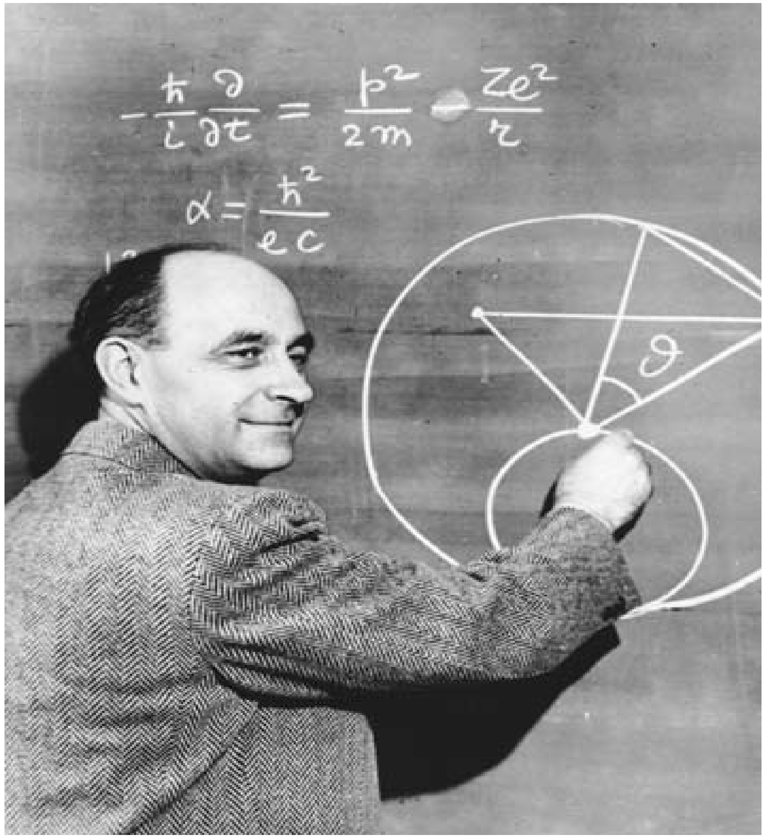

$\mathcal{L} = -\frac{1}{4} \tilde{F}_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

The title 'Field Theory and EW standard Model' encompasses developments of the last 60-70 years!

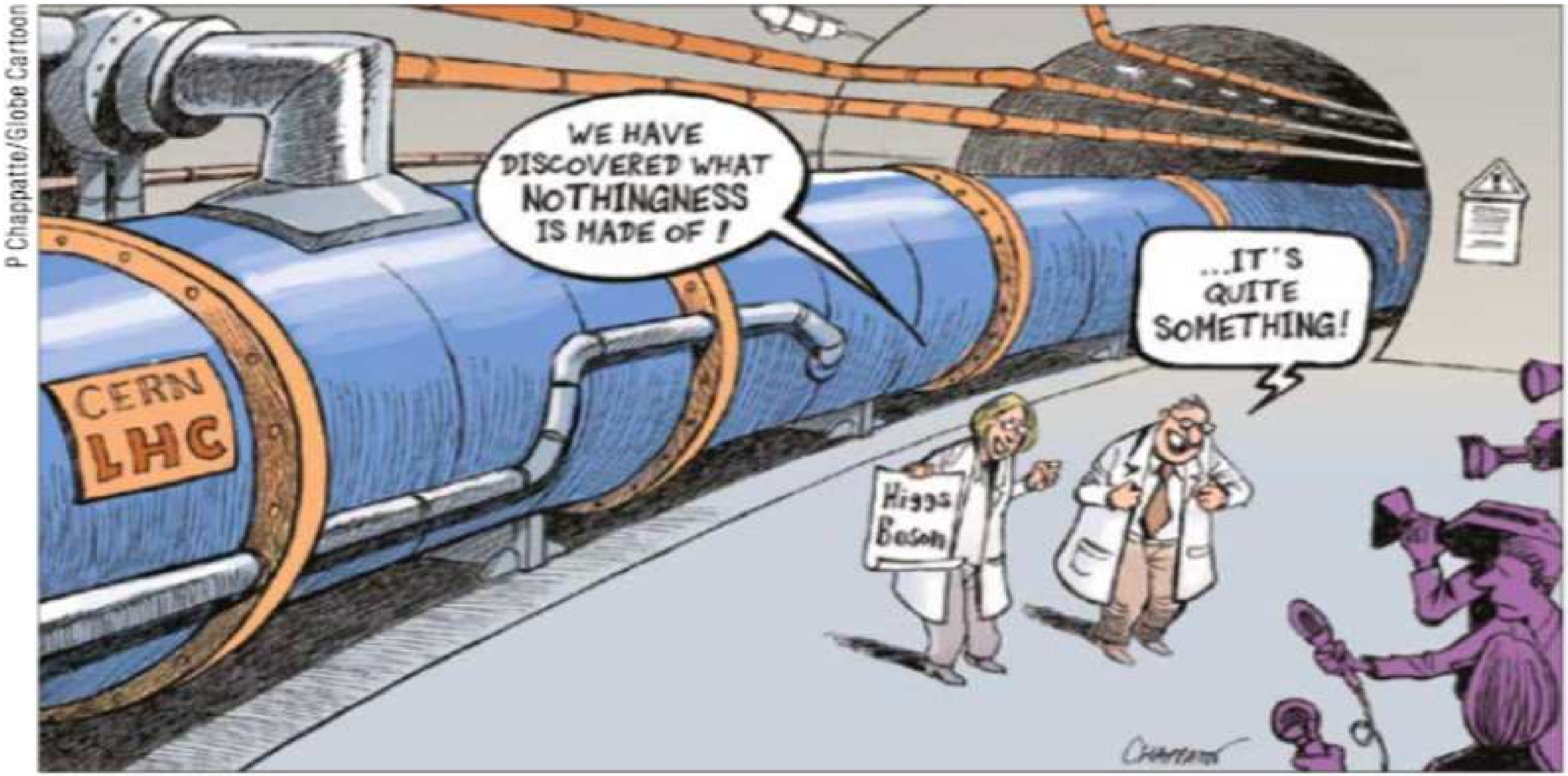
I am giving these lectures ten years after the discovery of the Higgs. This was the finale of the establishment of the correctness of the Standard Model as the correct theoretical description of EW interactions.

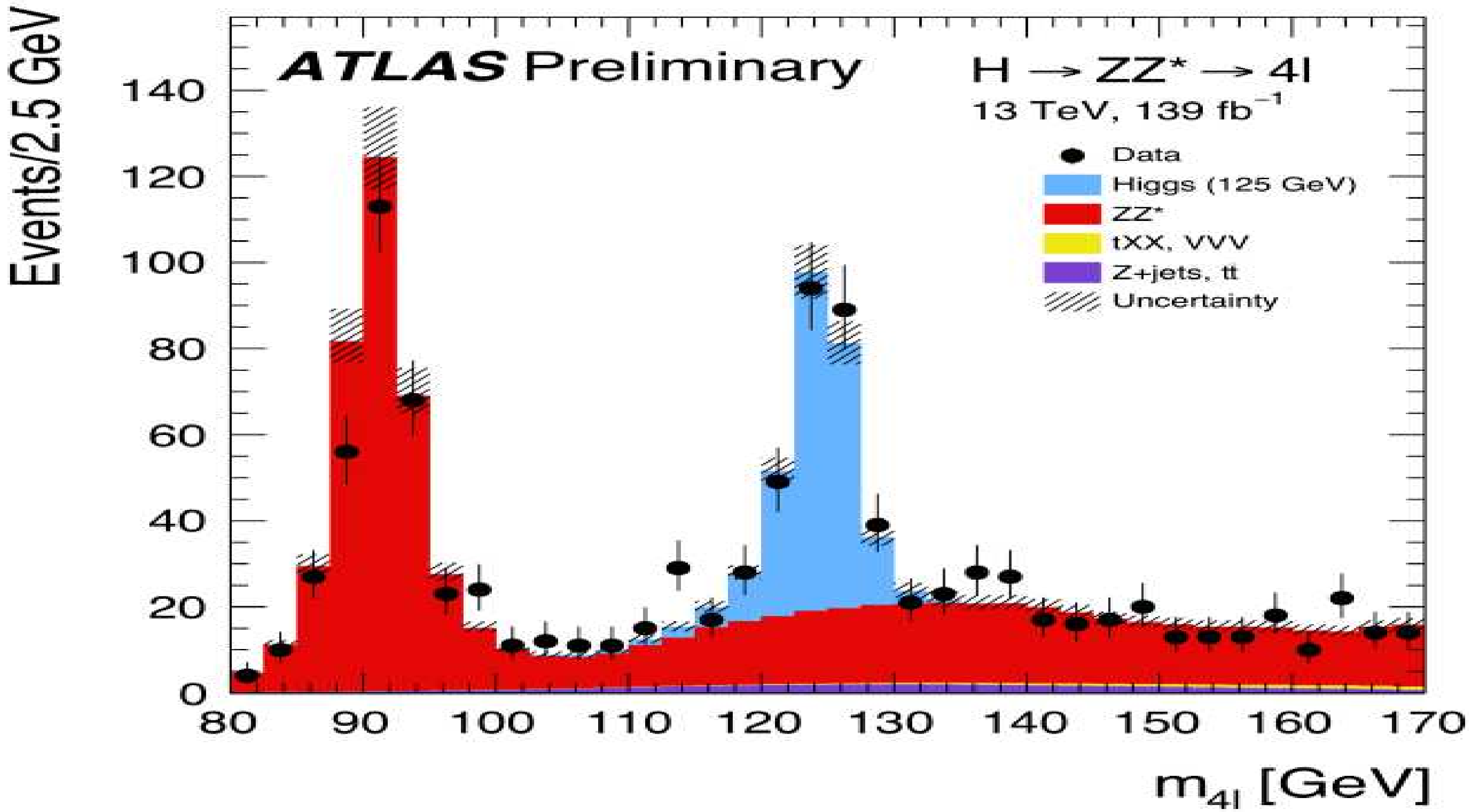
So I have had to think a little bit about the scope.



Enrico Fermi : β decay theory. 1933

Steven Weinberg : Model of Leptons 1967


WE HAVE DISCOVERED WHAT NOTHINGNESS IS MADE OF !
...IT'S QUITE SOMETHING!
CERN LHC
Higgs Boson
P Chappatte/Globe Cartoon



These are results from 2019!

All the data in collider and non collider experiments seem to confirm the predictions of the Standard Model, except a few chinks.

So knowing what we expect according to the SM is very important to find aberrations from it and hence probe way forward in understanding particle and their interactions and secrets of the Universe.

Aim of these lectures:

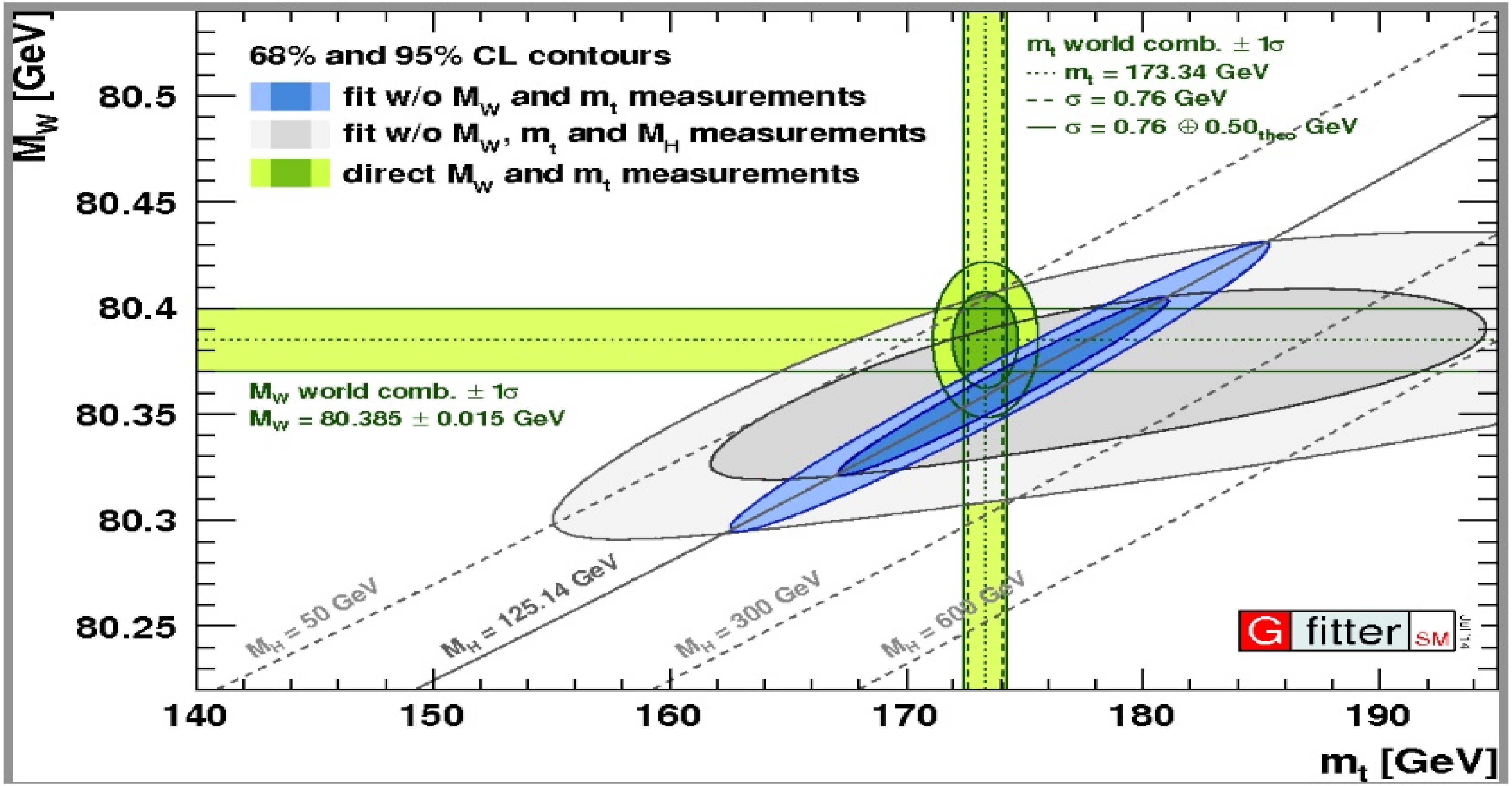
• An 'ode' to the SM!

• Point out salient and non negotiable aspects of EW phenomenology which helped establish the $SU(2)_L \times U(1)_Y$ gauge field theory as the correct theory of Electroweak interaction.

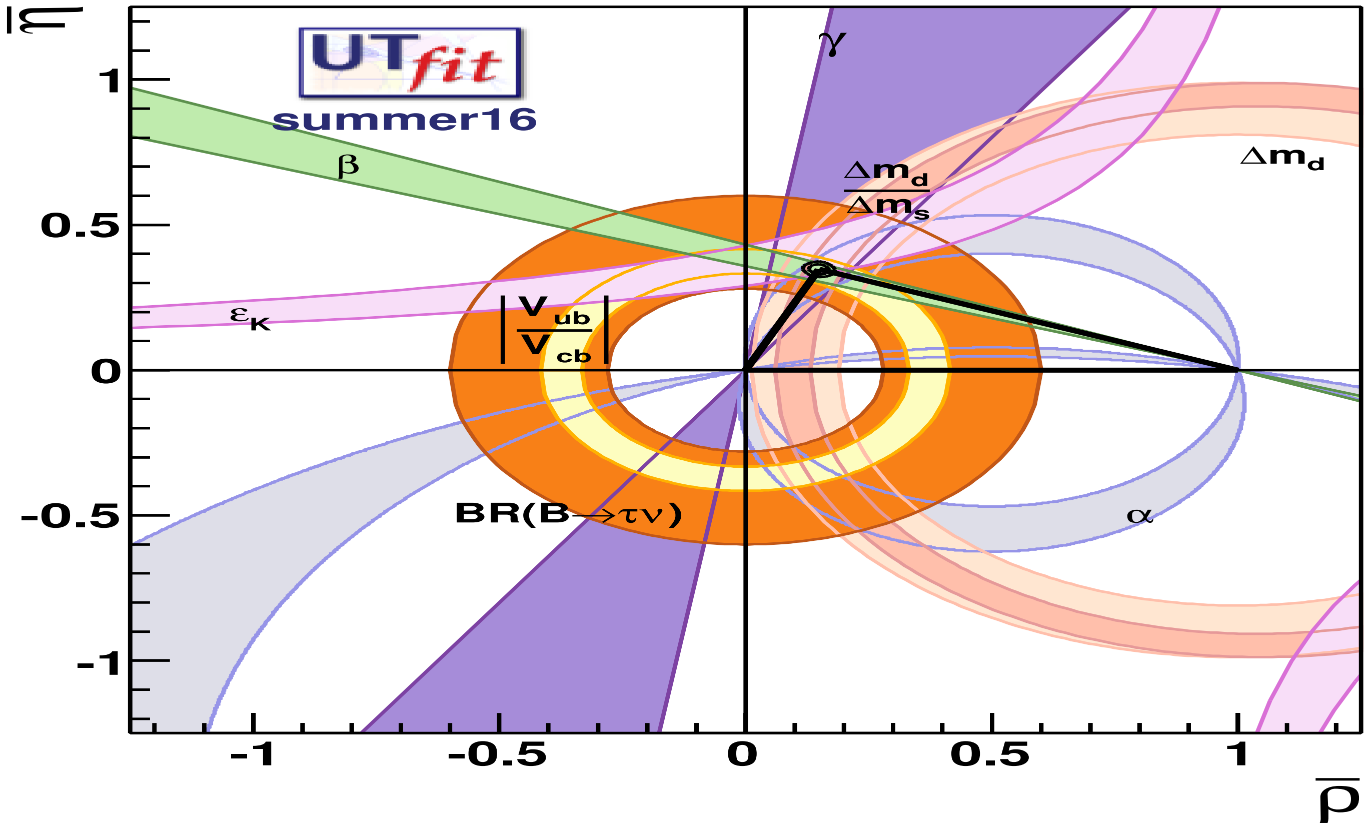
• SM *(ONLY the EW part)* as a Quantum Gauge Field Theory:

subtext: How consistency of EW theory itself pointed us to the missing parts and next goal posts!

• What is the new thing we learn about the SM **itself** from the Higgs discovery at the LHC!



SM rocks! At LOOP level. M_W slightly larger than the fit prediction.!



The three angles α, β and γ determined from host of experiments. According to the SM they should be angles of a triangle. The tip lies in the narrow area confirming that it is indeed the case

Lectures 1 and part of Lecture 2:

Mainly Physics of the Weak Bosons:

a) Setting up the notation of the SM Lagrangian, including Higgs mechanism

b) How one can understand the development of the SM also in terms of taming the bad high energy behavior of the scattering amplitudes!

c)The miracles of the particle spectrum of the SM: Anomaly cancellation and the Custodial symmetry!

Lecture 2 : Fermions, their couplings to W/Z in the SM and testing the SM at tree level.

Prediction of new particles and their masses in the SM:

a) Flavour mixing, CKM matrix

b) Flavour changing neutral current, GIM and all that

Prediction of M_c from the observed mass difference K_L–K_S. The 'first' use of an **indirect** effect to predict a mass!

c) Test of EW unification with the determination of $\sin\theta_w$ and resultant test of a unified gauge field theoretic description of Electro Weak interactions.

Lecture 3: SM development: A story of theoretical predictions for existence and masses of new particles being confirmed by experiments and vice versa.

a) Radiative corrections in a spontaneously broken gauge theory, oblique corrections and precision testing of the SM.

b) 'Indirect' determination of the mass of the top and Higgs!

c) Theoretical bounds on the Higgs mass

d) Implications of the measured mass of the Higgs for the SM! i.e the scale unto which SM can be consistent without any additional physics!

May not be able to cover c and d in detail but enough to vett your appetite.

1933: Fermi Theory of β decay in Modern Language:

$$n \to p + \nu_e + e. \;:\; \mathcal{H}_{int} = G_F\,(\bar{n}\gamma^\mu p)\,(\bar{\nu}^e\gamma_\mu e) + \text{h.c.}$$

$\Downarrow$

1957 : V -A Theory of weak decays by Sudarshan-Marshak and Feynman-Gell Mann:

$$n \to p + \nu_e + e\text{:}\;\; \mathcal{H}_{int} = \frac{G_F}{\sqrt{2}}\, C_V^\beta\, (\bar{n}\gamma^\mu(1 - 1.25\gamma_5)p)\,(\bar{\nu}^e\gamma_\mu(1-\gamma_5)e) + \text{h.c.}$$

$$\mu^- \to \nu_\mu + \bar{\nu}_e + e^- \;:\; \mathcal{H}^\mu_{int} = \frac{G_F}{\sqrt{2}}(\bar{\nu}^\mu\gamma^\mu(1-\gamma_5)\mu)\,(\bar{\nu}^e\gamma_\mu(1-\gamma_5)e) + \text{h.c.}$$

$\Downarrow$

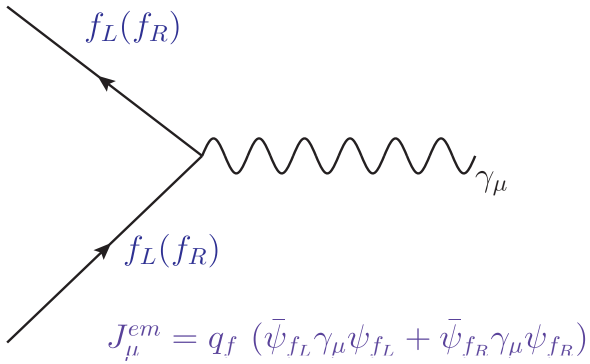
Weak Processes: Cabibbo, Kobayashi, Masakawa...(1964-1972)

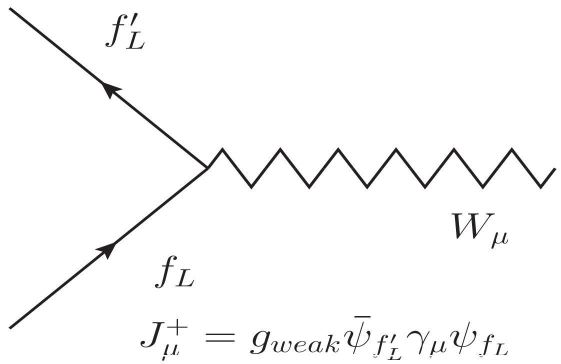
$$\mathcal{H}_{int} = \frac{G_F}{\sqrt{2}}\,\mathbf{V}_{ij}\,(\bar{u}_i\gamma^\mu(1-\gamma_5)d_j)\,(\bar{e}\gamma_\mu(1-\gamma_5)\nu^e) + \text{h.c.}$$

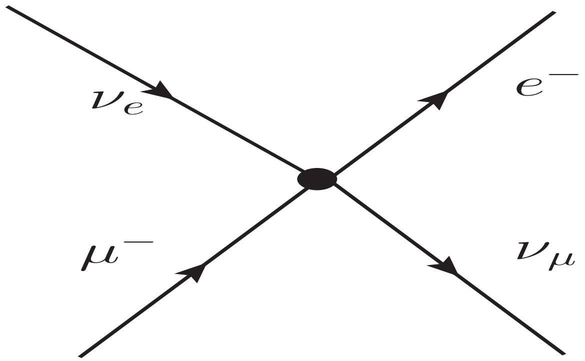
Existence of ONLY vector and axial vector currents means that the 'pointlike' four-fermion interaction can be thought of as being caused by an exchange of a spin 1 particle, the Weak boson.

(Introduced by Schwinger **before** the $V-A$ theory but mentioned by S-M in their paper.)

Extension of 'known' QED (Anticipating gauge theory here!):

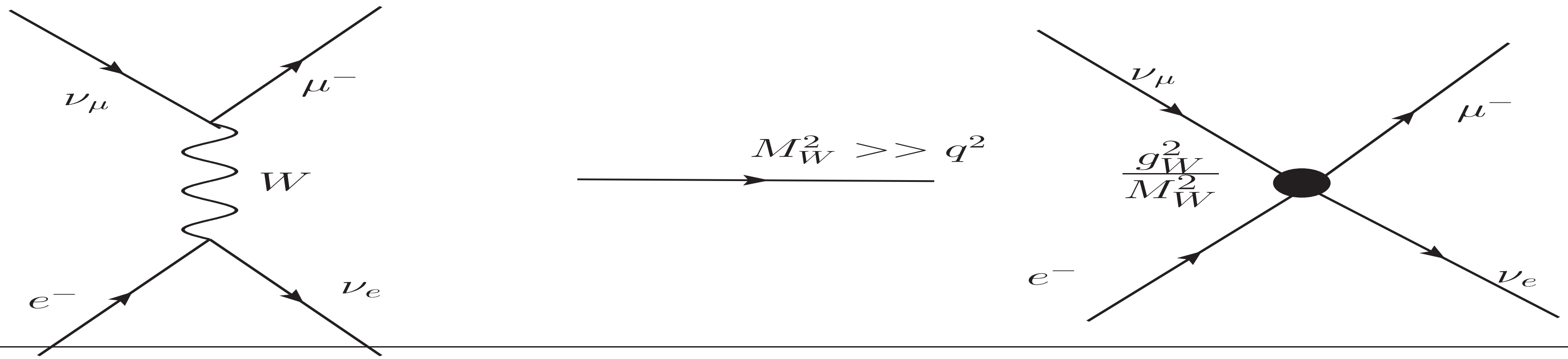






The cross-section for $\nu_\mu + e^- \to \nu_e + \mu^-$ calculated using contact interaction in the left rises with energy and violates unitarity for $\sqrt{s} \geq 300$ GeV. $\sigma_{tot} = \frac{G_F^2 s}{\pi} = \frac{2G_F^2 m_e E_{\nu_\mu}}{\pi}$. Same for $\sigma(\nu_e + n \to p + e^-)$.

Schwinger's suggestion: The violation of unitarity is avoided if the interaction is only 'effectively' pointlike and caused by the exchange of a heavy, massive spin 1 Boson.



Following the success of QED one wanted to have a gauge theory of Weak interactions.

Can one unify Electromagnetic and Weak interactions?

Problem: W had to be massive: short range and nonzero mass needed to preserve unitarity. Interacts with only the left handed fermions. Nonzero mass also breaks gauge invariance.

γ massless and interacts with left and right handed fermions. Its interactions described by a gauge theory which is renormalisable.

What is meant by a gauge theory? Very simply put it means that the interactions of matter particles with the force carrier gauge bosons and those of the gauge bosons among themselves are all controlled by the principle of Gauge Invariance.

The difference between the W and γ meant that for EW unification one needed to go beyond three gauge bosons. Why?

Recall: $\mathcal{H}^{weak}_{eff} = \frac{G_F}{\sqrt{2}}(J^W_\mu J^{W\mu\dagger}) = \frac{G_F}{\sqrt{2}}(\bar{\psi}_n\gamma_\mu(1-\gamma_5)\psi_p)(\bar{\psi}_{\nu_e}\gamma^\mu(1-\gamma_5)\psi_e)$

Or in general: $\equiv \frac{G_F}{\sqrt{2}}(\bar{\psi}_{3L}\gamma_\mu\psi_{1L})(\bar{\psi}_{4L}\gamma^\mu\psi_{2L})$

Here γ_μ are the Dirac γ matrices and $\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4$, with $1-\gamma_5 = P_L$, $1+\gamma_5 = P_R$ such that $P_{L,R}\psi = \psi_{L,R}$.

The charged current $J^{CC}_\mu = \bar{\psi}_{1L}\gamma_\mu\psi_{2L}$ and $J^{em}_\mu = \bar{\psi}_{1L}\gamma_\mu\psi_{1L}+\bar{\psi}_{1R}\gamma_\mu\psi_{1R}$.

So we have Massless γ which couples ot both L-chiral and R-chiral fermions with equal strength and two massive charged (weak) bosons which have interactions only with the left chiral fermion fields. This gives rise to L in $SU(2)_L$.

Only left-chiral fermions can belong to a non trivial representation of the weak gauge group. This means that Electromagnetic and Weak can not be unified using a simple gauge group.

SM -EW: $SU(2)_L \times U(1)_Y$ Gauge Field Theory describing Electro Weak (Electromagnetic and Weak) Interactions.

Thus SM stands on the joint pillars of relativistically invariant quantum field theories and gauge symmetries.

Gauge invariance guarantees renormalizability.

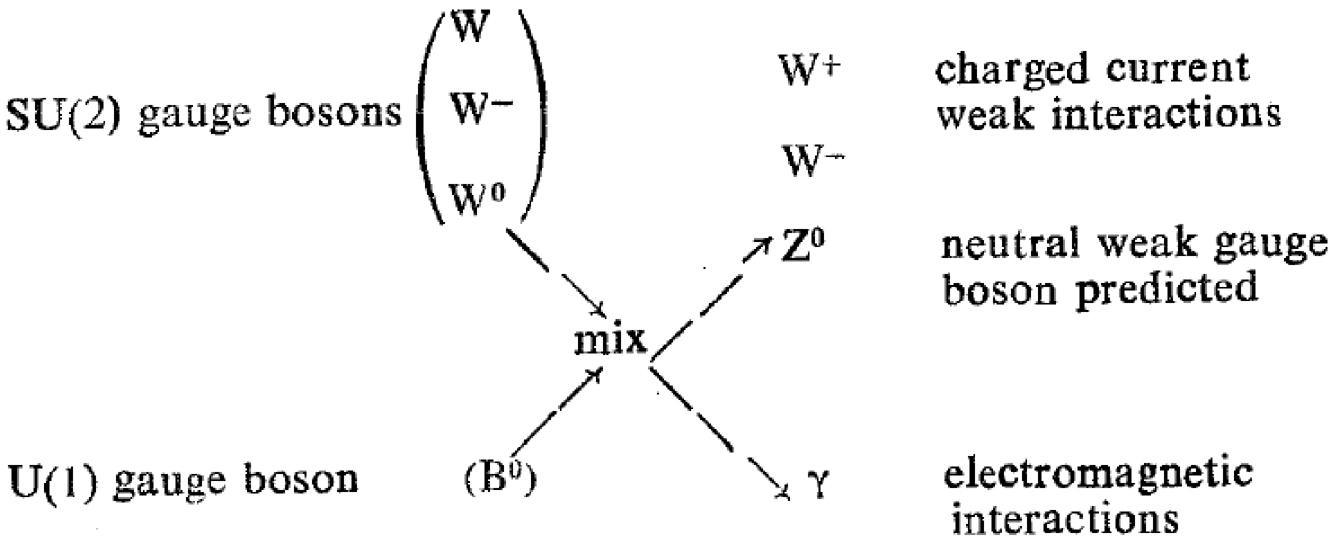
Spontaneous symmetry breaking of course allows us to have massive gauge bosons with spontaneous breaking of the $SU(2)_L \times U(1)_Y$ Gauge symmetry.

Its success, culminating in the discovery of the Higgs Boson.

Glashow implemented idea of Unification. No thought about how to generate nonzero masses for the W/Z

Weinberg on the other hand in his 1967 paper had the idea of unification and also used the Higgs mechanism to generate masses for the W/Z.

In the 1971 paper Weinberg included the quarks as well. So it is almost the final version of the EW part of the SM.



Glashow's paper Nucl.Phys. 22 (1961) 579-588. had EW unification but no 'model' for the nonzero masses of the weak bosons and hence no prediction for relation between m_W and m_Z. Much before the Higgs/Englert/Brout papers.

A MODEL OF LEPTONS*

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Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 17 October 1967)

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite[1] these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum. However, this raises the specter of unwanted massless Goldstone bosons.[2] This note will describe a model in which the symmetry between the electromagnetic and weak interactions is spontaneously broken, but in which the Goldstone bosons are avoided by introducing the photon and the intermediate-boson fields as gauge fields.[3] The model may be renormalizable.

and on a right-handed singlet

$$R = [\tfrac{1}{2}(1-\gamma_5)]e. \qquad (2)$$

The largest group that leaves invariant the kinematic terms $-\bar{L}\gamma^\mu\partial_\mu L - \bar{R}\gamma^\mu\partial_\mu R$ of the Lagrangian consists of the electronic isospin $\vec{T}$ acting on L, plus the numbers N_L, N_R of left- and right-handed electron-type leptons. As far as we know, two of these symmetries are entirely unbroken: the charge $Q = T_3 - N_R - \tfrac{1}{2}N_L$, and the electron number $N = N_R + N_L$. But the gauge field corresponding to an unbroken symmetry will have zero mass,[4] and there is no massless particle coupled to N,[5] so we must form our gauge group out of the electronic isospin $\vec{T}$ and the electronic hyperchange $Y = N_R + \tfrac{1}{2}N_L$.

Therefore, we shall construct our Lagrangian out of L and R, plus gauge fields $\vec{A}_\mu$ and B_μ coupled to $\vec{T}$ and Y, plus a spin-zero dou

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu + g\vec{A}_\mu\times\vec{A}_\nu)^2 - \tfrac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \bar{R}\gamma^\mu(\partial_\mu - ig'B_\mu)R - L\gamma^\mu(\partial_\mu ig\vec{t}\cdot\vec{A}_\mu - i\tfrac{1}{2}g'B_\mu)L$$

$$-\tfrac{1}{2}|\partial_\mu\varphi - ig\vec{A}_\mu\cdot\vec{t}\varphi + i\tfrac{1}{2}g'B_\mu\varphi|^2 - G_e(\bar{L}\varphi R + \bar{R}\varphi^\dagger L) - M_1{}^2\varphi^\dagger\varphi + h(\varphi^\dagger\varphi)^2. \qquad (4)$$

Physical Review Letters 27, 1688 (1971).

$$
\begin{aligned}
\mathfrak{L}' = {} & \frac{ig}{(g+g'^2)^{1/2}}[gZ^\nu - g'A^\nu][W^\mu(\partial_\mu W_\nu{}^\dagger - \partial_\nu W_\mu{}^\dagger) - W^{\mu\dagger}(\partial_\mu W_\nu - \partial_\nu W_\mu) + \partial^\mu(W_\mu W_\nu{}^\dagger - W_\nu W_\mu{}^\dagger)] \\
& - \frac{g^2}{(g^2+g'^2)} W_\mu W_\nu{}^\dagger (gZ_\rho - g'A_\rho)(gZ_\sigma - g'A_\sigma)(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma}) + \frac{g^2}{2}[|W_\mu W^\mu|^2 - (W_\mu W^{\mu\dagger})^2] \\
& + F(\varphi) - \frac{m_e}{\lambda}\varphi\bar{e}e - \tfrac{1}{8}(\varphi^2 + 2\lambda\varphi)[(g^2+g'^2)Z_\mu Z^\mu + 2g^2 W_\mu W^{\mu\dagger}] \\
& + i(g^2+g'^2)^{-1/2}\bar{e}\gamma^\mu\left[\left(\frac{1-\gamma_5}{2}\right)g'^2 + \frac{1}{2}\left(\frac{1+\gamma_5}{2}\right)(g'^2 - g^2)\right] eZ_\mu + \frac{i}{2}(g^2+g'^2)^{1/2}\bar{\nu}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)\nu Z_\mu \\
& + \frac{igg'}{(g^2+g'^2)^{1/2}}\bar{e}\gamma^\mu e A_\mu + \frac{ig}{\sqrt{2}}\bar{\nu}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)eW_\mu{}^\dagger + \frac{ig}{\sqrt{2}}\bar{e}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)\nu W_\mu . \qquad (1)
\end{aligned}
$$

The 'unified' description of EW interactions as a 'gauge' theory possible and it predicts a heavy, weak, electrically neutral boson Z and a new type of weak interaction (neutral weak interaction) mediated by this boson.

The incompatibility of nonzero masses of the gauge bosons with gauge invariance and hence renomalisability was sorted out by the Higgs mechanism.

Weinberg's 'model' of leptons predicted mass of the W and the Z, at tree level in terms of one free parameter of the theory.

1971 paper included the quarks as well.

Physical Review Letters 27, 1688 (1971).

(A) First write down a Lagrangian obeying some exact gauge symmetry, in which massless Yang-Mills fields interact with a multiplet of scalar fields' and other particle fields.

(B) Choose a gauge in which all the scalar field components vanish, except for a few (in our case one) real scalar fields.

(C) Allow the gauge group to be spontaneously broken by giving the remaining scalar field a nonvanishing vacuum expectation value. Redefine this field by subtracting a constant A. , so that the "shifted" field has zero vacuum expectation value. In the resulting perturbation theory, all vector mesons acquire a mass, except for those (in our case,the photon) associated with unbroken symmetries.

Free field Lagrangians for massless fermions and gauge bosons:

Real scalar: $\mathcal{L}_{scalar} = 1/2\partial_\mu\phi\partial^\mu\phi - 1/2m^2\phi^2$;

Complex scalar: $\mathcal{L}_{c.scalar} = \partial_\mu\phi^*\partial^\mu\phi - m^2|\phi|^2$;

Massless fermion: $\mathcal{L}_f = i\bar{\psi}\gamma^\mu\partial_\mu\psi$

Abelian Gauge Field: $\mathcal{L}_{gauge} = -1/4F_{\mu\nu}F^{\mu\nu}$, with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, Nonabelian Gauge Field:

$\mathcal{L}_{nobabelian} = -1/4 F^a_{\mu\nu} F^{a,\mu\nu}$, with $F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{abc} W^c_\mu W^c_\nu$

where $f^{abc} = \epsilon^{abc}$ for $SU(2)$ gauge group.

In general

$[T^a, T^b] = i f^{abc} T^c$

With T^a being the generators and f^{abc} being called the structure constants.

For $SU(2)$ gauge group $T^a = \sigma^a/2$ where σ^a are Pauli matrices.

The important aspect in gauge theories is that given a representation of the gauge group to which the fermions belong gauge invariance completely dictates the form of the interaction.

So the ∂_μ in $\mathcal{L}_f$ above gets changed to the covariant derivative D_μ, which for the Electromagnetic case is

$$D_\mu = \partial_\mu - ieA_\mu$$

$$i\bar{\psi}\gamma^\mu D_\mu \psi = \bar{\psi}\gamma^\mu \partial_\mu \psi + ie\bar{\psi}\gamma^\mu \psi A_\mu$$

One can see easily that this will give rise to $\mathcal{L}_{int} = +J^{em}_\mu A^\mu$ with $J^{em}_\mu = \bar{\psi}\gamma_\mu \psi$

In general if Φ represents a matter field (spin 1/2 or spin 0) transforming according to a representation T_{IJ} of the gauge group then

$$D_\mu \Phi_I = \partial_\mu \Phi_I + igV_\mu (T)_{IJ} \Phi_J$$

with

$$V_\mu = W^a_\mu, A_\mu, B_\mu.$$

Then the matter part of the Lagrangian is invariant under a local gauge transformation provided the gauge field also transforms as

$$W^a_\mu \to W^a_\mu + 1/g \partial_\mu \alpha^a + f^{abc} W^a_\mu \alpha^c$$

with $\Phi_I \to exp^{-ig(T)^a_{IJ} \alpha_a(x)} \;\; \Phi_J$

The current that one will get for the $SU(2)_L$ gauge group for a pair of Fermions belonging to the doublet representation will have exactly the same form as J_μ^+ We will write these tomorrow.

The mass term for the gauge bosons $M_V^2 V^{a\mu} V_\mu^a$ is clearly non invariant under these gauge transformations.

If ψ_{fL} and ψ_{fR} should belong to different representations of the gauge group the fermion mass term will also break gauge invariance.

$$m_f \bar{\psi}_f \psi_f = m_f (\bar{\psi}_{fL} \psi_{fR} + \bar{\psi}_{fR} \psi_{fL})$$

Since the right-chiral fermions do NOT have interactions with W, the ψ_L and ψ_R have to belong to different representations of the gauge group.

Gauge Group	Gauge Boson Fields	Coupling
$SU(2)_L$	W^a_μ, $a = 1, 2, 3$	g_2
$U(1)_Y$	B_μ	g_1

The gauge charge for $U(1)_Y$ is Y. (Recall for $U(1)$ this normalization is free, not so for nonabelian gauge groups)

Gauge transformation for matter fields:

$$\psi \to exp^{-ig_1/2\alpha_Y(x)}\psi$$

Quarks	Leptons
$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$
$u_R,\ c_R,\ t_R$ $d_R,\ s_R, b_R$	$e_R,\ \mu_R,\ \tau_R$
+anti-quarks	+ anti-leptons

Colour indices suppressed. In the 'Standard version' of the SM strictly no right handed neutrinos. $\mathcal{Q}_{iL}$ and $\mathcal{L}_{iL}$ are the quark and lepton doublets with $i = 1, 2, 3$ standing for the three generations. $d, s, , b$ correspond to mass eigenstates. Y for $\mathcal{Q}_i = 1/3$, Y for $\mathcal{L}_i = -1$.

In the context of weak interactions it is more correct to write :

Quarks	Leptons
$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$

$$\begin{pmatrix} d'_1 \\ d'_2 \\ d'_3 \end{pmatrix}_L \equiv \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \mathbf{V}_{ij} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

$\mathbf{V}_{ij}$ are the elements of the CKM matrix which we will come to in tomorrow's lecture.

The essence of unification is in Glashow's observation:

$Q = T_{3L} + Y/2$ (Q is the electromagnetic charge in units of $|e|$, where e is electron charge.)

where $Y = 1/3$ for quark doublets and $Y = -1$ for lepton doublet. Thus Y is the hyper charge of the $U(1)$ symmetry group. i.e the electromagnetic charge is made up of T_{3L} and Y. In turn this means that the A_μ will have to be a linear combination of W_μ^3 and B_μ.

Right handed fermions are singlets under $SU(2)_L$ and hence a different Y value than the left handed fermions and is simply $2Q$

This clearly means that along with the gauge bosons the mass term for the fermions too is not invariant under gauge transformations.

Thus the net symmetry is $SU(2)_L \times U(1)_Y$ g_2 coupling constant for $SU(2)$ and g_1 for $U(1)_Y$.

$SU(2)_L$: Three gauge bosons W^1, W^2, W^3 : all couple only to left handed fermions.

$U(1)_Y$: B couples to all fermions (LH and RH).

B and W^3 mix, giving one zero mass eigenstate γ. Identify other with a new Neutral vector boson called Z.

$$\mathcal{M}^2_{GB} = \begin{pmatrix} m^2_{W^1} & 0 & 0 & 0 \\ 0 & m^2_{W^2} & 0 & 0 \\ 0 & 0 & m^2_{W^3} & m_{WB} \\ 0 & 0 & m_{WB} & m^2_B \end{pmatrix}$$

Diagonalise the neutral gauge boson mass matrix demanding that $m_\gamma = 0$. This give the mixing angle in terms of the $m^2_{W^3}, m^2_B$.

$$A_\mu = cos\theta_w B_\mu + \sin\theta_w W^3_\mu$$

$$Z_\mu = -\sin\theta_w B_\mu + \cos\theta_w W^3_\mu$$

$e = g_2 \sin\theta_w = g_1 \cos\theta_w$ and $\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{e^2}{8M_W^2 \sin^2\theta_w}$

We will derive the last two relations in the next lecture.

$SU(2)_L \times U(1)$ model predicts

1)New particles: Z, charm quark c.

2)Weak Neutral current mediated by Z analogous to Weak Charged Current mediated by the W.

3)One parameter θ_w, decides the couplings of the various particles to the Z. **These are decided by the gauge group and representation of the gauge group to which fermions belong!**

4)WWZ coupling possible and predicted.

5)A sort of unification: Unless $\sin\theta_W$ small (then the whole idea is not sensible) e, g_1, g_2 all of similar order. The difference in strengths of interactions only apparent due to large M_W.

6)If $g_2 = e$ then $M_W \sim 100$ GeV.

7)Since $G_F = \frac{g_2^2}{8M_W^2}$ and e, g_2 related through $\sin\theta_w$, $e = g_2 \sin\theta_w$

demanding

$|\sin\theta_w| < 1$ already gives $M_W > 37.4$GeV.

This is the content of the Glashow paper as well as Weinberg/Salam papers

Start with

$$\mathcal{L} = \mathcal{L}_{gauge}^{massless} + \mathcal{L}_{f}^{massless}$$

$$\mathcal{L}_{f}^{massless} = i\bar{\mathcal{Q}}_{iL} D_\mu \gamma^\mu \mathcal{Q}_{iL} + i\bar{u}_{iR} D_\mu \gamma^\mu u_{iR} + ... i\bar{\mathcal{L}}_{iL} D_\mu \gamma^\mu \mathcal{L}_{iL} + i\bar{l}_{iR} D_\mu \gamma^\mu l_{iR}$$

where i in $\mathcal{Q}_i$, u_{iR} etc. stands for generations. $\mathcal{Q}_i, \mathcal{L}_i$ are the doublets written in the table earlier l_{iR} are the charged lepton fields.

As argued above mass terms for both the gauge bosons and fermions will spoil the gauge invariance.

We want gauge invariance as it guarantees the renormalisability (for example the Ward Takahashi identities for QED or the Slavnov-Taylor identities for nobabelian gauge field theories).

This is where the Higgs mechanism which breaks the gauge symmetry spontaneously comes into play. The vacuum breaks gauge symmetry but the scattering amplitudes obey the relations implied by the gauge invariance.

The fermion couplings with the gauge bosons are those dictated by the gauge symmetry. Recall, for example, the covariant derivative $D_\mu = \partial_\mu - ieA_\mu$

Higgs mechanism:

Now start from a color singlet, complex scalar field which is a $SU(2)_L$ doublet, call it Φ and take Φ to have $Y = 1$.

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \Re e\phi^0 + i\Im m\,\phi^0 \end{pmatrix}$$

where $\phi_i = \Re e(\phi_i) + i\Im m(\phi_i)$. Thus we have four real scalar fields. Y_ϕ the hypercharge is taken to be 1.

$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi - V(\Phi)$$

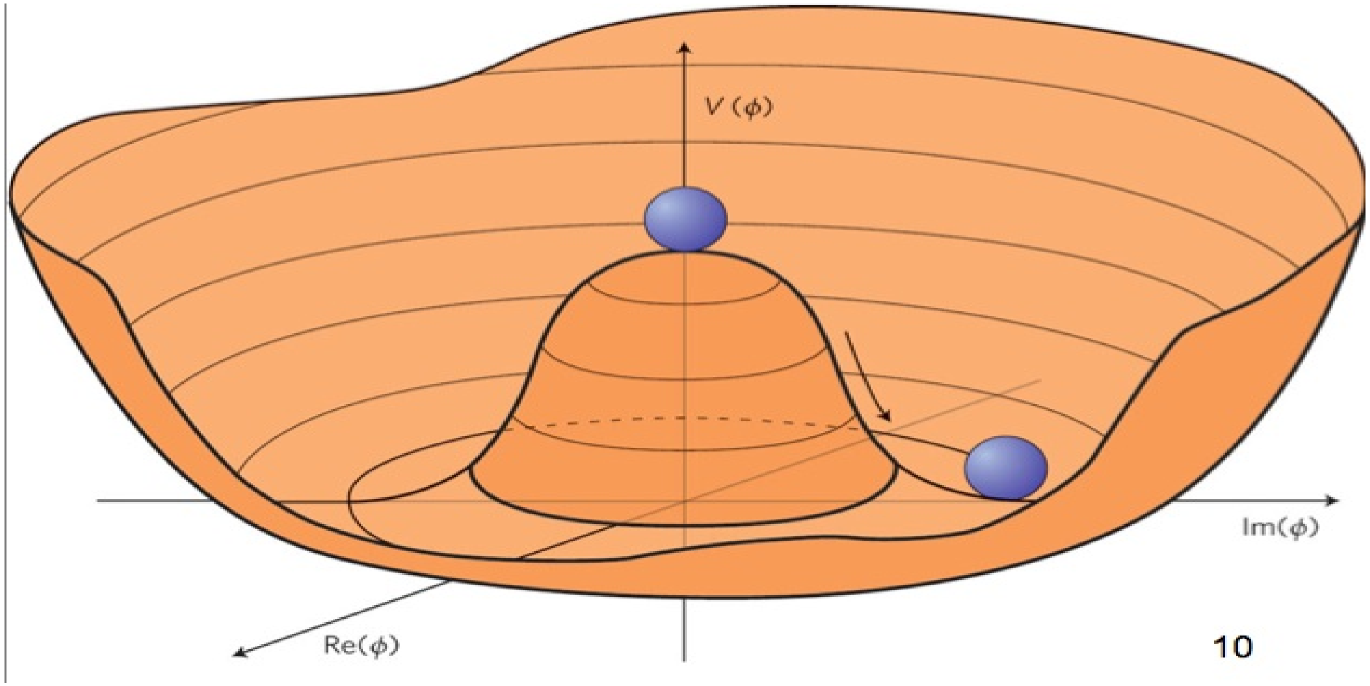
where the $V(\phi)$ has to have the very specific form to cause the spontaneous symmetry breaking.

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D_\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2$$

with $\mu^2 < 0$.

Compared to the Lagrangian for a complex scalar we wrote initially this has the wrong sign for the quadratic term. So μ is not the mass.

But it is precisely this wrong sign that is required for the spontaneous symmetry breaking to occur.



The SSB occurs when the vacuum is such that the scalar field has a non vanishing vacuum expectation value.

The choice of direction in the ϕ_1, ϕ_2 space where the field acquires the nonzero vev decides what would be the unbroken symmetry

The choice of the representation of the Higgs field decides pattern of symmetry breaking.

Inherent in Glashow's partial symmetry with $Q = T_{3L} + Y/2$ is the requirement that $U(1)$ gauge invariance corresponding to the Electromagnetism is left unbroken.

I.e out of the four scalar fields only the electromagnetically neutral scalar field can receive nonzero vev.

The required symmetry breaking pattern is guaranteed (choice $Y_\Phi = 1$)

$$< 0|\Phi|0 > = < \Phi >_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

with $v = \sqrt{\frac{-\mu^2}{\lambda}}$. For negative μ^2 clearly v is real.

$SU(2)_L \times U(1)_Y \to U(1)_{em}$.

The three scalar degrees of freedom corresponding to the three Goldstone bosons in fact disappear from the spectrum in the Unitary Gauge and the three gauge bosons $W_\mu^{1,2}$ and a linear combination of B_μ, W_μ^3 become massive. The mass terms arising from the covariant derivative terms for the complex scalar!

Now when one rewrites the theory in terms of fluctuations around this minimum configuration

$$\Phi(x) = \begin{pmatrix} \theta_2 + i\theta_1 \\ v/\sqrt{2} + h(x)/\sqrt{2} - i\theta_3 \end{pmatrix}$$

$$= exp(i\theta^a \sigma_a / v) \begin{pmatrix} 0 \\ v/\sqrt{2} + h(x)/\sqrt{2} \end{pmatrix}$$

where σ^a are the Pauli spin matrices. One recognizes the factor outside as that for a gauge transformation for a $SU(2)_L$ doublet.

So one can go to a gauge where

$$\Phi'(x) = \begin{pmatrix} 0 \\ v/\sqrt{2} + h(x)/\sqrt{2} \end{pmatrix}$$

Consider the term: $(D_\mu\Phi)^\dagger(D_\mu\Phi)$ and substitute the above expression for Φ' . This is the unitary gauge.

$$\mathcal{L}_{VVM} = \frac{g_2^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{(g_1^2+g_2^2)v^2}{8} Z_\mu Z^\mu + 0 A_\mu A^\mu$$

with

$$A_\mu = cos\theta_w B_\mu + \sin\theta_w W_\mu^3$$

$$Z_\mu = -\sin\theta_w B_\mu + \cos\theta_w W_\mu^3$$

and

$\tan\theta_w = g_1/g_2$ with $e = g_2 \sin\theta_w = g_1 \cos\theta_w$

Now the new thing from what I wrote before when I did not have a model how to get masses in a gauge invariant way, now M_W, M_Z are related.

$M_Z^2 = M_W^2/\cos\theta_w$, with

$M_W^2 = g_2^2 v^2/8$, $v = \left(\frac{1}{\sqrt{2}G_F}\right)^{1/2} \simeq 246$ GeV.

$\frac{M_W^2}{M_Z^2 \cos^2\theta_w} = 1 = \rho$

This ρ is in fact the ratio of strength of the neutral current interactions to charged current interactions in the 'current-current' description of the fermion-fermion interaction via Z and W exchange, in the limit $M_W, M_Z \to \infty$.

This was found to be close to 1 from measurements before W, Z were found!

Another observation:

This relationship is the result of the choice of the doublet representation for the Higgs field

and

can be understood in terms of an accidental symmetry that this potential seems to have, called Custodial Symmetry.

Now when rewrites the $\mathcal{L}_\Phi$ in the unitary gauge we have

$$\mathcal{L}_h = 1/2(\partial_\mu h)^2 + \mu^2 h^2 - \lambda v h^3 - \lambda/4 h^4$$

A real scalar field, with mass $2\mu^2$.

Since $v = \sqrt{(} - \mu^2/\lambda)$ the mass of the Higgs boson is given in terms of self coupling λ.

This is an arbitrary parameter of the Higgs potential, not fixed by any condition.

We will come back to this later when we look at limits on the Higgs mass!

Thus the SSB creates a mass term for the gauge bosons from the covariant derivative term which is Gauge invariant any way.

Thus in the unitary gauge now the propagating degrees of freedom are

3 massive gauge bosons $W^{\pm}, Z$ and one massless gauge boson γ and ONE propagating massive scalar

So out of the four scalar degrees of freedom only one is left and the other three provide the degrees of freedom corresponding to the longitudinal polarization necessary for the massive gauge boson.

The particle spectrum after the Spontaneous symmetry breaking is easiest seen in the unitary gauge.

$\mathcal{L}_{gauge}^{massless}$	$+\ \mathcal{L}_\Phi$	SSB	$\mathcal{L}_{gauge}^{massive}$	$+\ \mathcal{L}_h$
4 massless gauge bosons	4 scalar fields		3 massive, 1 massless gauge bosons	1 physical scalar

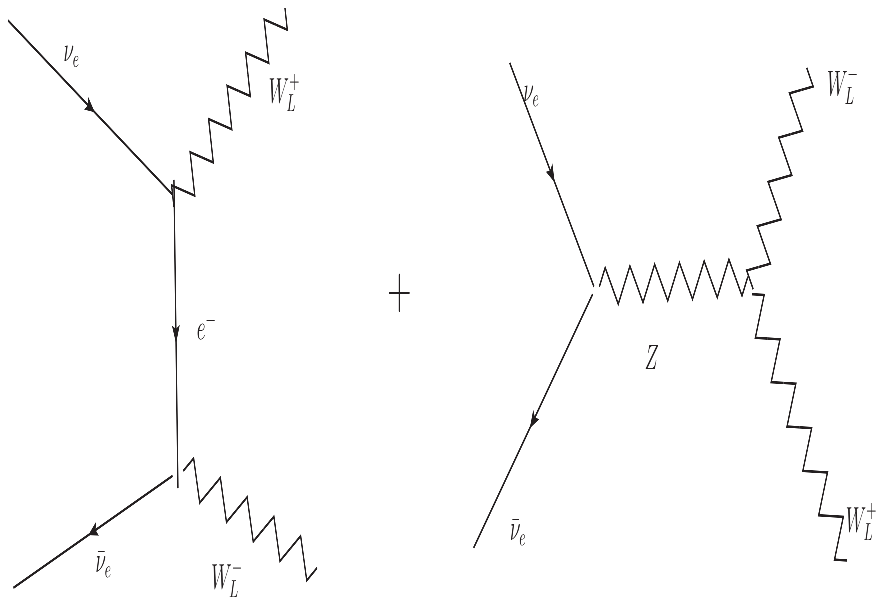
Count the degrees of freedom and you will see it is 12 both before and after the SSB.

While we will write the couplings of the W, Z with fermions in the next lecture let us note that the charged current with fermions will be exactly the same as J_μ^{CC}. I use it in the discussion in the next slide.

1. Fermi theory (Current-Current Interactions): $\mathcal{M}(\nu e \to \nu e)$ violates tree level unitarity for $\sqrt{s} \simeq 250 \sim G_F^{-1/2}$ GeV $\Rightarrow$ massive gauge bosons. Mass? Somewhere below this!

2. Even with a mass for the IVB, $\mathcal{M}(\nu_e \bar{\nu}_e \to W^+ W^-)$ grows too fast with energy and violates unitarity.

3. S channel exchange of a Z boson in gauge theory with precisely the non abelian gauge couplings of $SU(2)_L \times U(1)$ gauge group restores the unitary behaviour. Divergence for $WW \to WW$ MUCH worse, also cured!.



J.S. Bell: Nuclear Physics, **B60**, 427, 1973:

Showed that in a renomalisable theory tree level amplitudes satisfy unitarity.

But three sets of authors asked the opposite question: What can we deduce by demanding that tree level amplitudes satisfy unitarity.

J. M. Cornwall, D. N. Levin and G. Tiktopoulos, PRL **30**, 1268 (1973), Phys.Rev. **D10**, 1145 (1974),

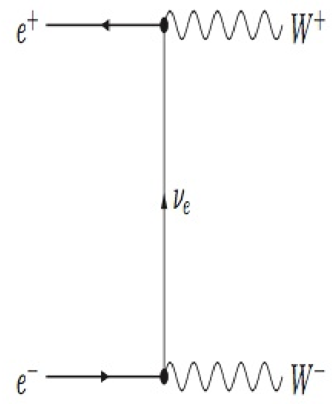
C. Llewllyn Smith : PLB **46**, 233 (1973)

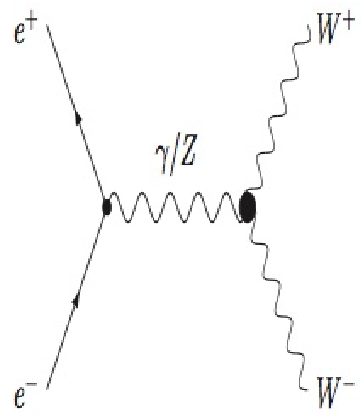
S.D. Joglekar,: Ann. Phys. **83**, 427 (1974)

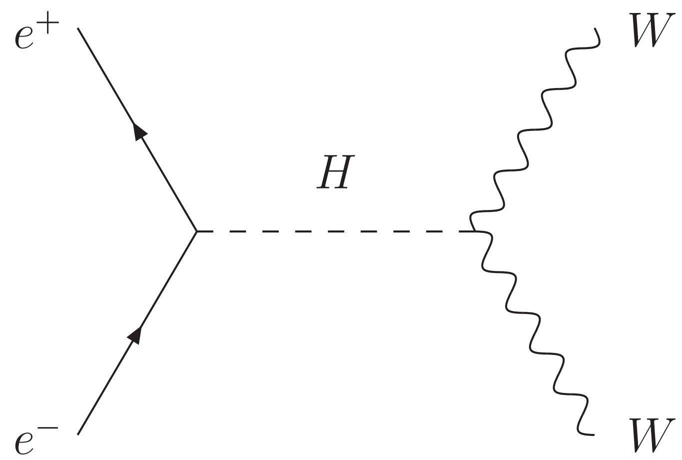
They showed that such demands uniquely indicate spontaneously broken gauge theories.

For example:

Unitarity of $\mathcal{A}(e^+e^- \to W_L W_L)$, for example, $\Rightarrow$ Divergences cancel only if there is a $J = 0$ amplitude (s channel exchange of a Spin 0 particle) whose coupling to matter/gauge particles is proportional to their masses $\Rightarrow$ **Existence of a Higgs boson.**







But no knowledge on the scale! ie. the mass of the Higgs!

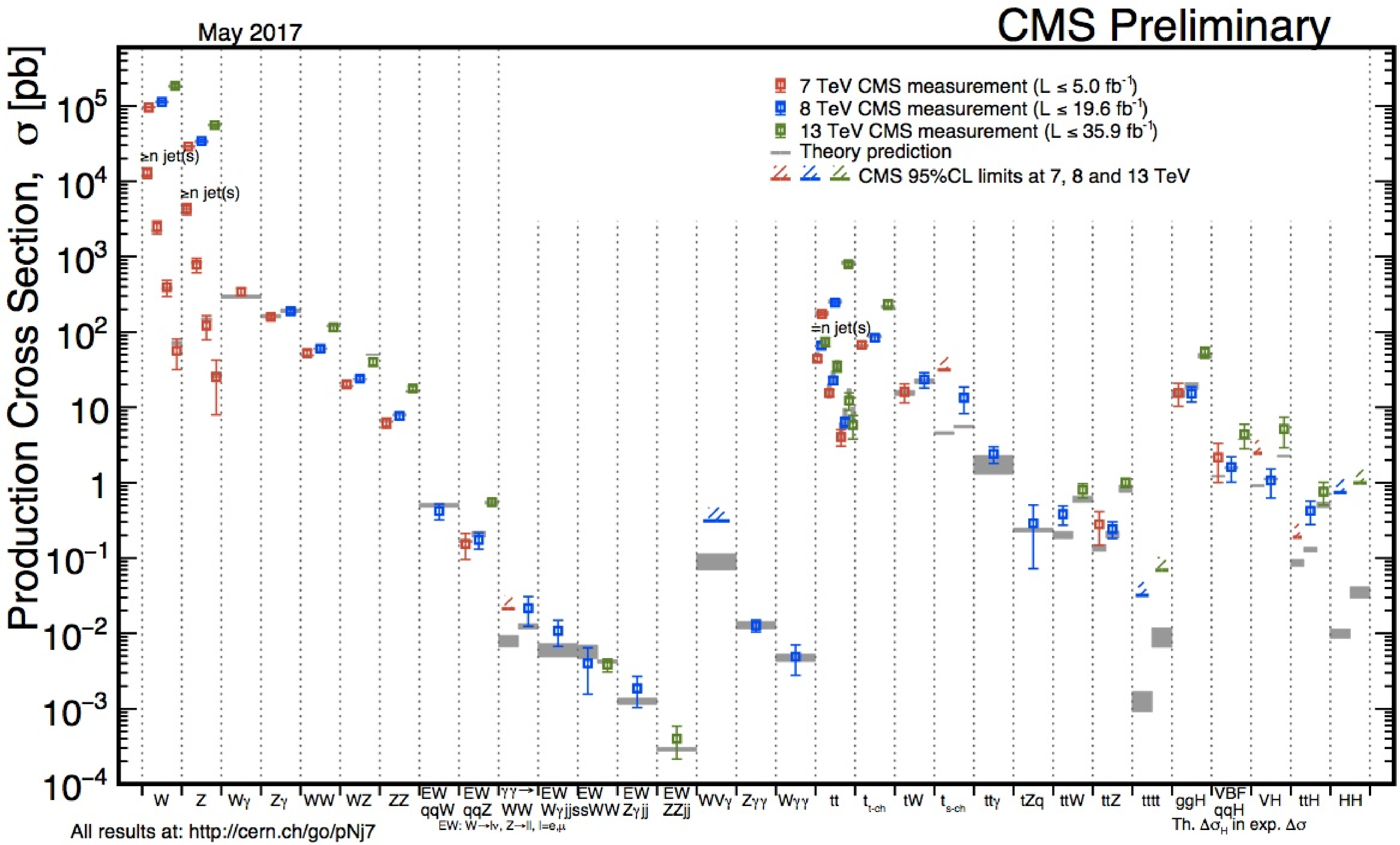
Only the couplings.

SSB Gauge theory makes scattering amplitudes well behaved at high energy, even with massive gauge bosons!.

In fact Higgs couplings to matter and gauge bosons required to be proportional to masses to have unitarity.

This is indeed one of the prediction of the renormalisable $SU(2)_L \times U(1)_Y$ where the EW symmetry broken spontaneously by the Higgs mechanism!

Additional slides


CMS Preliminary
May 2017
Production Cross Section, σ [pb]
7 TeV CMS measurement (L ≤ 5.0 fb⁻¹)
8 TeV CMS measurement (L ≤ 19.6 fb⁻¹)
13 TeV CMS measurement (L ≤ 35.9 fb⁻¹)
Theory prediction
CMS 95%CL limits at 7, 8 and 13 TeV
≥n jet(s)
≥n jet(s)
=n jet(s)
W Z Wγ Zγ WW WZ ZZ EW qqW EW qqZ γγ→WW EW Wγjj ssWW EW Zγjj EW ZZjj WVγ Zγγ Wγγ tt t-ch tW ts-ch ttγ tZq ttW ttZ tttt ggH VBF qqH VH ttH HH
EW: W→lν, Z→ll, l=e,μ
Th. ΔσH in exp. Δσ
All results at: http://cern.ch/go/pNj7

The 'Periodic Table' of Fundamental particles and their interactions has arrived!

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W$^-$	80.39	–1
W$^+$ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

The force carriers are fundamental particles too!

July 4, 2012

BOSONS spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Spin $\neq$ 1

Name	Mass GeV/c^2	Electric Charge
H^0	about 125	0

H^0 (Higgs Boson)

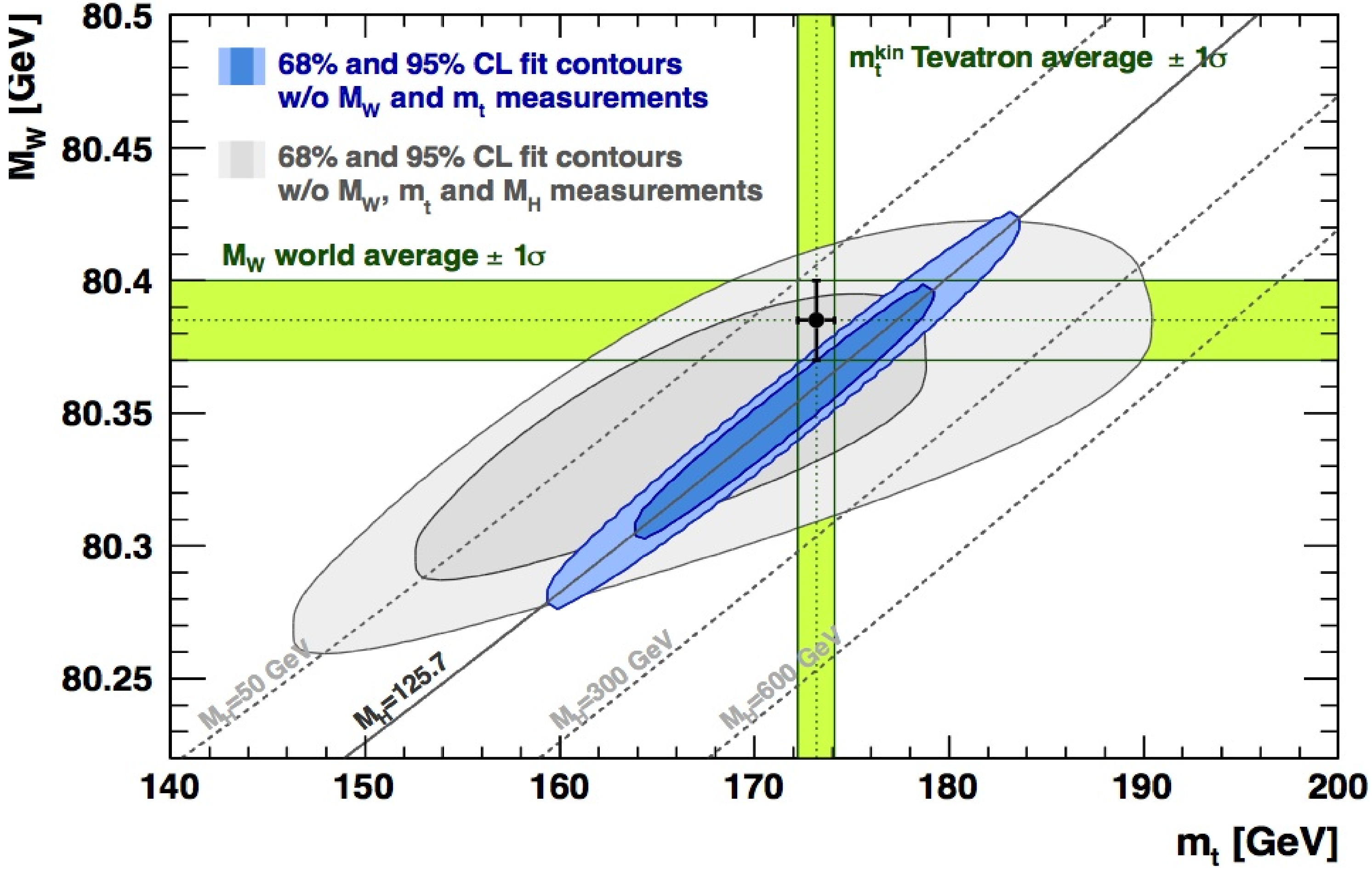
The observed signal is called a Higgs Boson in the following, although its detailed properties and in particular the role that the new particle plays in the context of electroweak symmetry breaking need to be further clarified. The signal was discovered in searches for a Standard Model (SM)-like Higgs. See the following section for mass limits obtained from those searches.

H^0 MASS

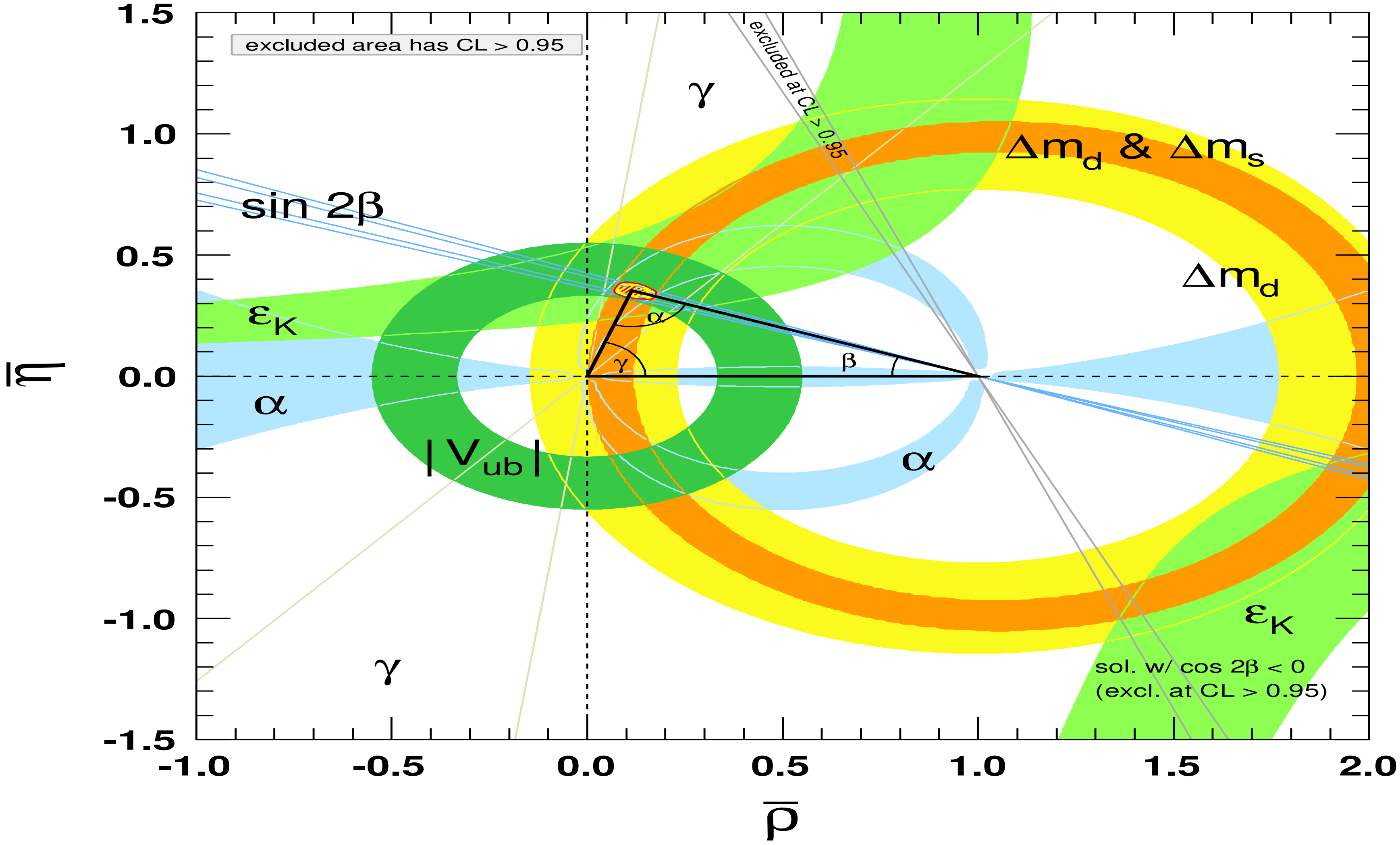
VALUE (GeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
125.9±0.4 OUR AVERAGE				
125.8±0.4±0.4	[1] CHATRCHYAN	13J	CMS	pp, 7 and 8 TeV
126.0±0.4±0.4	[2] AAD	12AI	ATLS	pp, 7 and 8 TeV
• • • We do not use the following data for averages, fits, limits, etc. • • •				
126.2±0.6±0.2	[3] CHATRCHYAN	13J	CMS	pp, 7 and 8 TeV
125.3±0.4±0.5	[4] CHATRCHYAN	12N	CMS	pp, 7 and 8 TeV

HTTP://PDG.LBL.GOV Page 1 Created: 7/31/2013 15:05

2013 Update of the PDG!



SM rocks! **LOOP Level**!



No theory predictions for the mixing in the quark sector!

Not all the 'matter' fermions were discovered accidentally.

Existence for some and even their masses (c, t) was predicted in a renormalisable gauge theory: for example for the charm it was the suppression of the FCNC and the small mass difference between K_L and K_S. This is the part where the properties of a quantum field theory were used crucially.

Not just this, existence of some members of a lepton and quark family, combined with the requirement of anomaly cancellation meant that the remaining members had to exist. The t and the ν_τ were hunted for very actively once the b and the τ made their appearance!

The necessity of massive gauge bosons was indicated by demanding the internal consistency of the effective description of the weak processes as a current current interaction.

The 'unified' description of EW theories as a 'gauge' theory predicted a new type of weak interaction (neutral weak interaction) and also a heavy, weak, electrically neutral boson.

What is meant by a gauge theory? Very simply put it means that the interactions of matter particles with the force carrier gauge bosons and those of the gauge bosons among themselves are all controlled by the principle of Gauge Invariance.

The existence of an universal $V - A$ theory of weak interactions and purely vector nature of the electromagnetic interactions predicted existence of the 'Z'!

The incompatibility of nonzero masses of the gauge bosons with renomalisability was sorted out by the Higgs mechanism.

Weinberg's 'theory' of leptons predicted mass of the W and the Z, at tree level in terms of one free parameter of the theory.

The Higgs was needed to give masses to the gauge bosons in a gauge invariant way!

Theory needed existence of the Higgs but was mum on the mass of the Higgs, which was a free parameter of the theory just like the $\sin\theta_w$ was for the unified theory!

Till in the end one could also bound the Higgs mass in an 'indirect' manner from the precision measurements, as well as from the more 'theoretical' arguments!

1) g-2 of the muon

2) Flavour non universality

3)Precision measurement of the mass of the W.

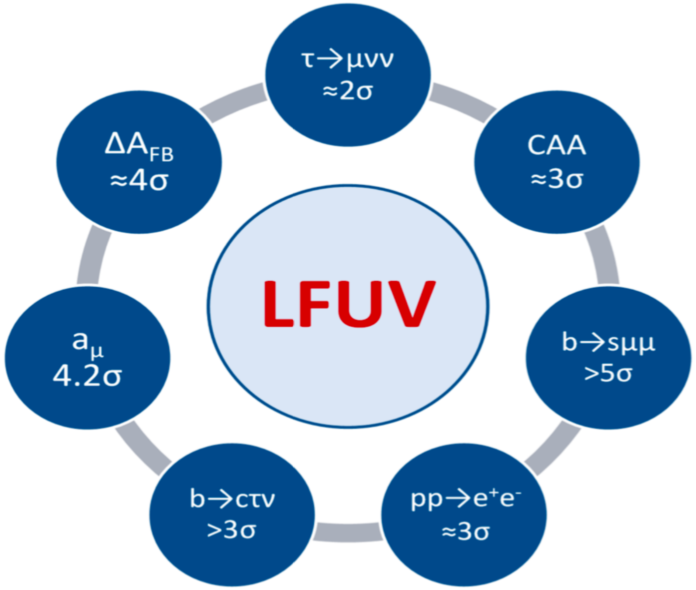
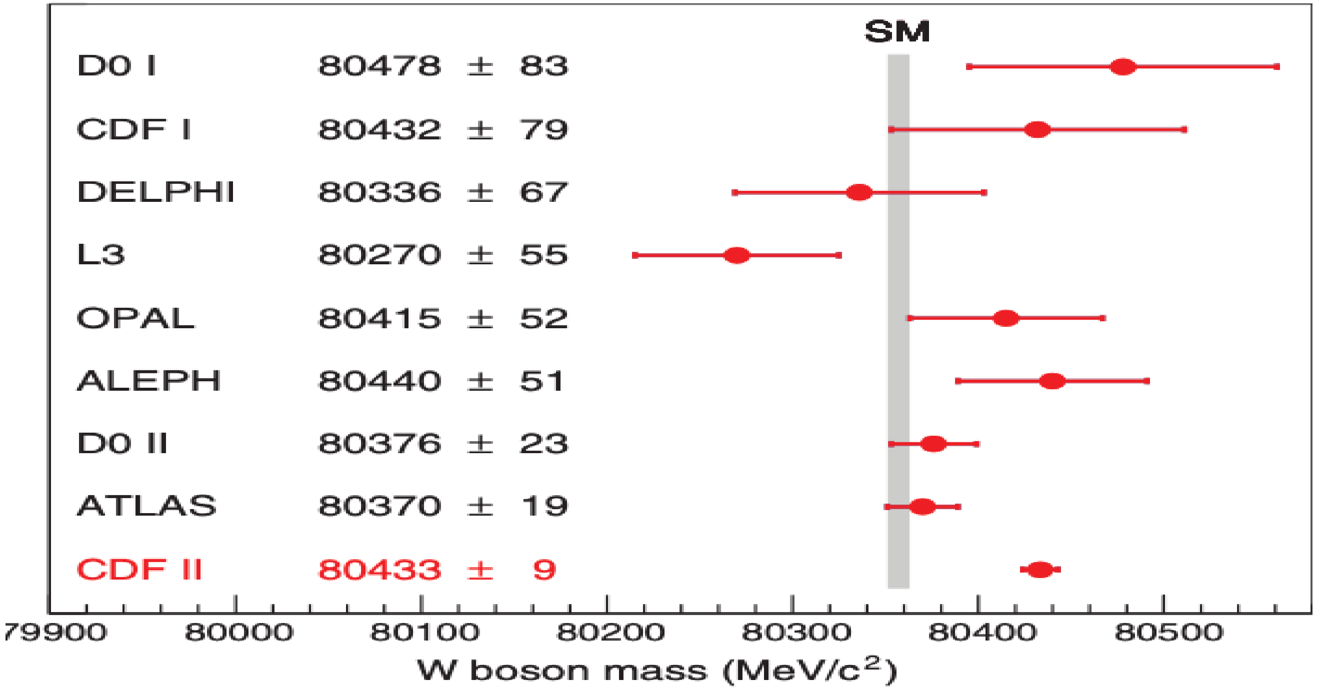
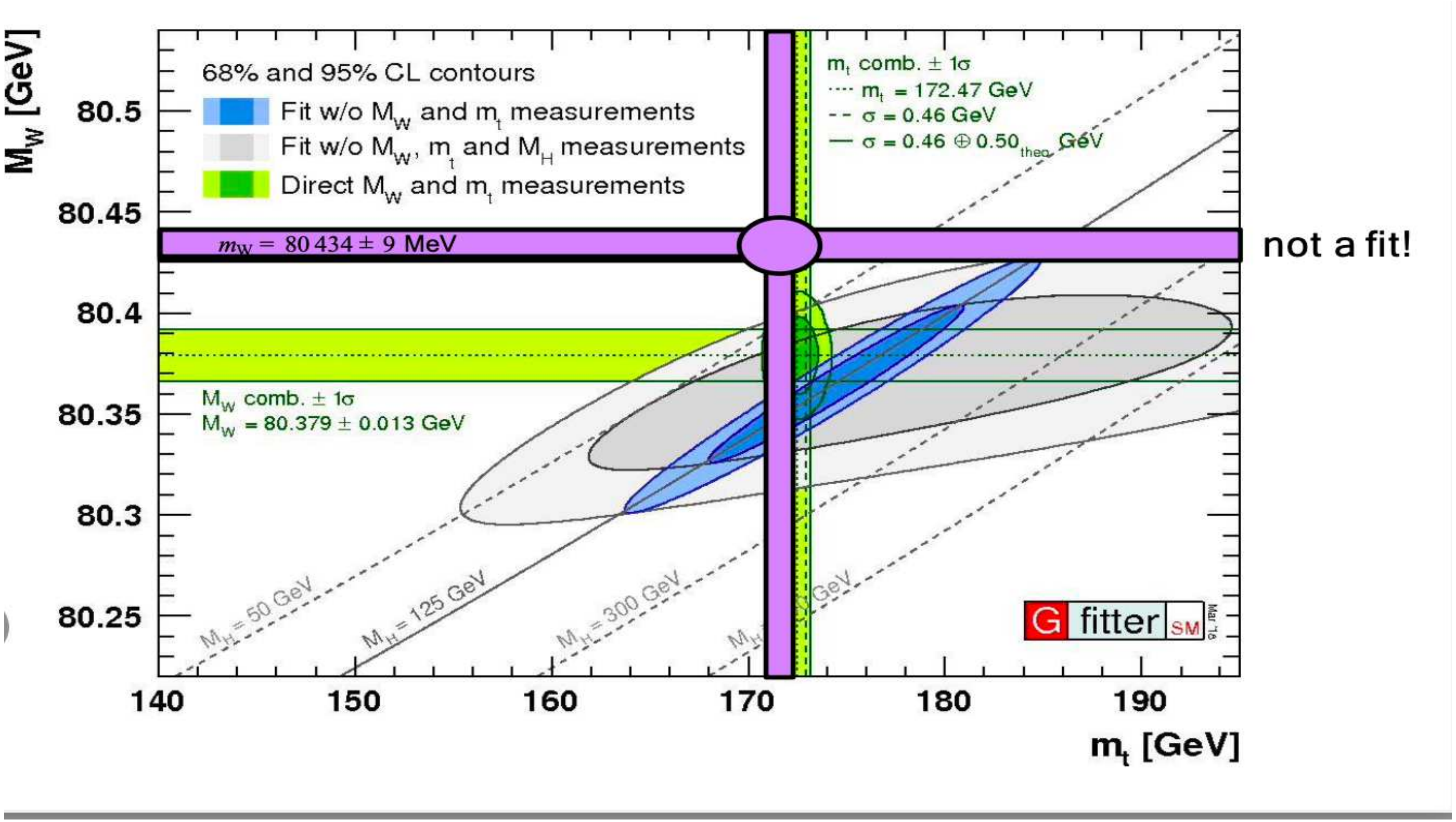


Figure 1: Summary of the experimental hints for LFUV beyond the SM.

Lepton Flavour Universality Violation: is that the thin edge of the wedge? Ref: Unveiling Hidden Physics at the LHC [arXiv:2109.06065 [hep-ph]].



In this case it is the precision of the new experimental result that needs scrutiny!



(From Martijn Mulders).

Difference between the theoretical expectation in the SM and measured value at one ppb. (Phys. Rev. Lett. 126, 141801). Evidence for new physics seems compelling. Critical evaluation of theory 'prediction' necessary.

[2104.03281]