Field Theory and EW Standard Model

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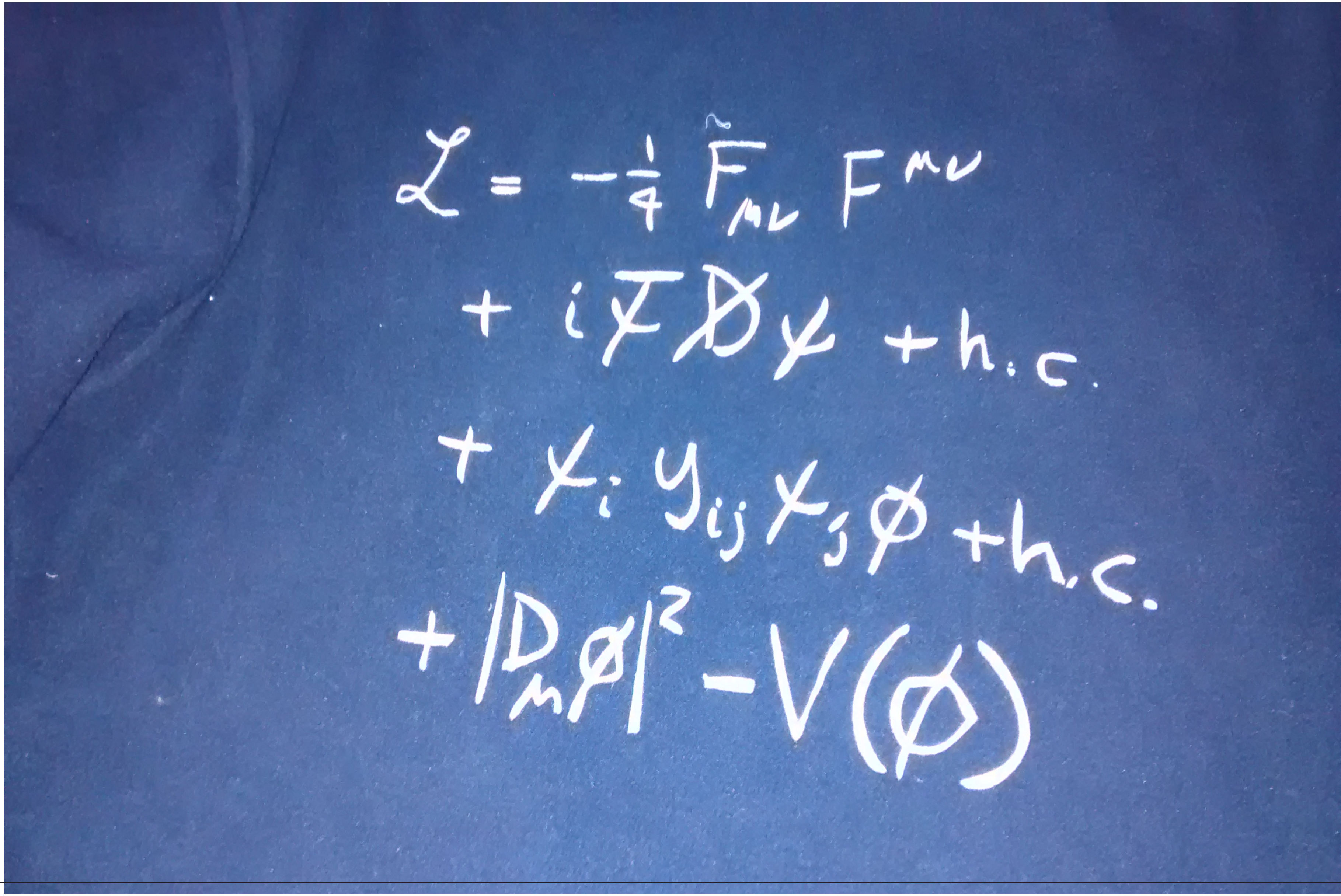

$\mathcal{L} = -\frac{1}{4} \tilde{F}_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

Lectures 1 and part of Lecture 2:

Mainly Physics of the Weak Bosons:

a) Setting up the notation of the SM Lagrangian, including Higgs mechanism

b) How one can understand the development of the SM also in terms of taming the bad high energy behavior of the scattering amplitudes!

c)The miracles of the particle spectrum of the SM: Anomaly cancellation and the Custodial symmetry!

Lecture 2 : Prediction of new particles and their masses in the SM. Fermions, their couplings to W/Z in the SM and testing the SM at tree level.

a) Flavour mixing, CKM matrix

b) Flavour changing neutral current, GIM and all that

Prediction of M_c from the observed mass difference K_L–K_S. The 'first' use of an **indirect** effect to predict a mass!

c) Test of EW unification with the determination of $\sin\theta_w$ and resultant test of a unified gauge field theoretic description of Electro Weak interactions.

"Model of Leptons" : 1967 Weinberg Paper.

Number of parameters of the EW sector of the SM:

$SU(2)$ coupling g_2, $U(1)$ couplings g_1, vev v , λ and μ^2.

EW symmetry breaking condition relates v, λ and μ^2.

The parameters are then g_2, g_1, v and λ.

Masses of all the bosons in the theory controlled by these four.

$$M_W = \frac{v}{2} g_2, \quad M_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2} = \frac{M_W}{\cos\theta_W}, M_h = \sqrt{2\lambda}v.$$

where

$$\tan\theta_w = \frac{g_1}{g_2}, e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}, \sin\theta_w = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} \Rightarrow v^2 = \frac{1}{\sqrt{2}G_\mu} \Rightarrow v \simeq 250\,\text{GeV}.$$

We can trade g_2, g_1, v for $G_F(G_\mu), \sin\theta_w$ and e.

We get

$$M_W = 2^{-5/4} G_F^{-1/2} \frac{e}{\sin\theta_W} \simeq \frac{37}{\sin\theta_W}\,\text{GeV}$$

This gives us the prediction:

$$M_W > 37 \quad \text{GeV} \quad , \quad M_Z > M_W$$

At this stage M_W, M_Z depend on $G_\mu, \sin\theta_w$ and e.

1) Couplings of the gauge bosons with fermions are dictated by the gauge group Symmetry.

2)Couplings of the gauge bosons with each other are also dictated by the gauge group Symmetry.

3)Couplings of the fermions and gauge bosons with the higgs are of course decided by the SSB, essentially the representation of the Higgs.Symmetry breaking

Even though the SM was put forward in 1967 (with quarks in 1971) it was begun to be taken seriously only in 1974!

Theory reason: Renormalisability was proved by 't Hooft and Veltman.

Observational: The expected W, Z masses were way above the energies available then. So there was no hope of finding evidence for them 'directly'. Was it possible to find this evidence 'indirectly?'

Of course one could look for processes which can take place via mediation of a Z boson.

$\nu_\mu + e^- \to \nu_\mu + e^-, \bar{\nu}_\mu + e^- \to \bar{\nu} + e^-$ (only via NC)

$\nu_\mu + e^- \to \nu_e + \mu^-$ (only via CC)

$\nu_e + e^- \to \nu_e + e^-, \bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$. (both via CC and NC)

In 1974 two things happened:

1)Evidence for neutral current interactions in $\nu_\mu + e^- \to \nu_\mu + e^-$ scattering in the BEBC.

2) Discovery of the charmonium, a bound state of c and $\bar{c}$ in November 1974 where the mass M_c seemed to be consistent with that expected from a gauge theory of weak interactions, viz. 1.6 GeV.

Followed by results on ν_μ scattering off the nuclei also gave evidence for $\rho = 1$ ie interaction responsible for the charged current and neutral current processes were of the same strength.

Both the discovery of the c quark with the mass it had and $\rho \simeq 1$ were indications that weak interactions were described by a gauge theory based on $SU(2)_L \times U(1)_Y$ where the symmetry was broken spontaneously by a Higgs doublet.

To discuss these we need to write down the interactions of the quarks and leptons with W/Z bosons.

Consider the kinetic term for fermions in the Lagrangian, $i\bar{\psi}_L \not\partial \psi_L + i\bar{\psi}_R \not\partial \psi_R$ where ∂_μ has been replaced by D_μ. Ψ_L is a doublet of two fermions like (u, d) or (ν_e, e^-) etc.

$$\partial_\mu \Psi_L \to D_\mu \Psi_L = \partial_\mu \Psi_L - i\frac{g_1 Y_\Psi}{2} B_\mu \Psi_L - ig_2 W_\mu^a \frac{\tau^a}{2} \Psi_L,$$

$$D_\mu f_R = \partial_\mu f_R - i\frac{g_1 Y_{f_R}}{2} B_\mu f_R.$$

This will lead to terms in addition to the regular $\mathcal{L}_{kin}^f$ and those are the interaction terms. Starting from

$$\sum_{i=1}^3 \left[i\mathcal{L}_L^i \not{D} \mathcal{L}_L^i + ie_R^i \not{D} e_R^i + i\mathcal{Q}_L'^i \not{D} \mathcal{Q}_L'^i + iu_R'^i \not{D} u_R'^i + id_R'^i \not{D} d_R'^i \right]$$

we get: $\Delta\mathcal{L}_{\text{int}} = (\frac{1}{2} g_1 J^{\mu\,Y} B_\mu + g_2 + J^{\mu\,3} W_\mu^3) + \frac{g_2}{2\sqrt{2}} (J^{\mu\,+} W_\mu^+ + J^{\mu\,-} W_\mu^-)$

Where,

$J^{\mu+} = 2(\bar{\nu}^i_L \gamma^\mu e^i_L + \bar{u}^i_L \gamma^\mu \mathbf{V}_{ij} d^j_L)$, $J^{\mu-} = (J^{\mu+})^\dagger$ same as J^{CC}_μ

$J^{\mu Y} = \sum_{f'} Y_{f'} \bar{f}' \gamma_\mu f'$ with $f' = f_L, f_R$ and $J^{\mu 3} = \sum_f T_{3L} \bar{f}_L \gamma_\mu f_L$

B couples to both f_L, f_R, W^3 couples to both f_L. Rewrite the terms involving neutral current in terms of mass eigenstates Z_μ, A_μ.

$$\left[\frac{1}{2} g_1 \cos\theta_W \, J^{\mu\, Y} + g_2 \sin\theta_W \, J^{\mu\, 3}\right] A_\mu + \left[-\frac{1}{2} g_1 \sin\theta_W \, J^{\mu\, Y} + g_2 \cos\theta_W \, J^{\mu\, 3}\right] Z_\mu$$

Since $J^{em}_\mu = \sum_f q_f (\bar{f}_L \gamma_\mu f_L + \bar{f}_R \gamma_\mu f_R)$ and $q'_f = \frac{Y'_f}{2} + T^{f'}_{3L}$, we must identify $e = g_1 \cos\theta_w = g_2 \sin\theta_w$.

$$\mathcal{L}^{NC} = g_Z J_\mu^Z$$

$$J_\mu^Z = g_L^f \bar{f}\gamma_\mu f_L + g_R^f \bar{f}\gamma_\mu f_R = \frac{1}{2} g_V^f \bar{f}\gamma_\mu f - \frac{1}{2} g_A^f \bar{f}\gamma_\mu \gamma_5 f$$

Important thing:

g_L^f and g_R^f decided by q_f and T_{3L} of the fermions.

This means it is exactly the same for all the fermions of a given charge q_f and a given T_{3L} independent of the generation.

$$g_L^f = T_3(f_L) - \sin^2\theta_w\ q_f, \quad g_V^f = T_3(f_L) + T_3(f_R) - 2\ q_f \sin^2\theta_w$$
$$g_R^f = T_3(f_R) - \sin^2\theta_w\ q_f, \quad g_A^f = T_3(f_L) - T_3(f_R)$$

f	ν	e^-	u	d
g_L^f	$\frac{1}{2}$	$-\frac{1}{2}+\sin^2\theta_W$	$\frac{1}{2}-\frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W$
g_R^f	0	$\sin^2\theta_W$	$-\frac{2}{3}\sin^2\theta_W$	$\frac{1}{3}\sin^2\theta_W$
g_A^f	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
g_V^f	$\frac{1}{2}$	$-\frac{1}{2}+2\sin^2\theta_W$	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$

$g_V^e \sim 0$ for $\sin\theta_w = 0.25$.

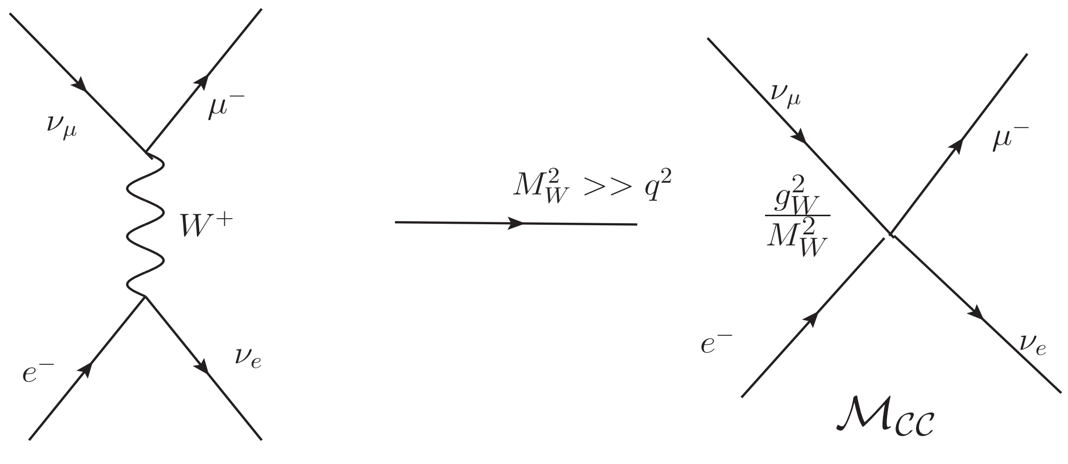
Charged current couplings include $\mathbf{V}_{ij}$, ie the W couples flavours across generations.

Weak neutral current couples fermions of the same charge and always of the same generation. No cross-generational or flavour changing weak neutral current.

We will take this up later when we discuss quark phenomenology further.

The W and Z exchange generate effective interaction Hamiltonian which can describe various charged current and neutral current processes.

Consider: $e^- + \nu_\mu \to \mu^- + \nu_e$ a charged current process



$g_W = \frac{g_2}{2\sqrt{2}}$.

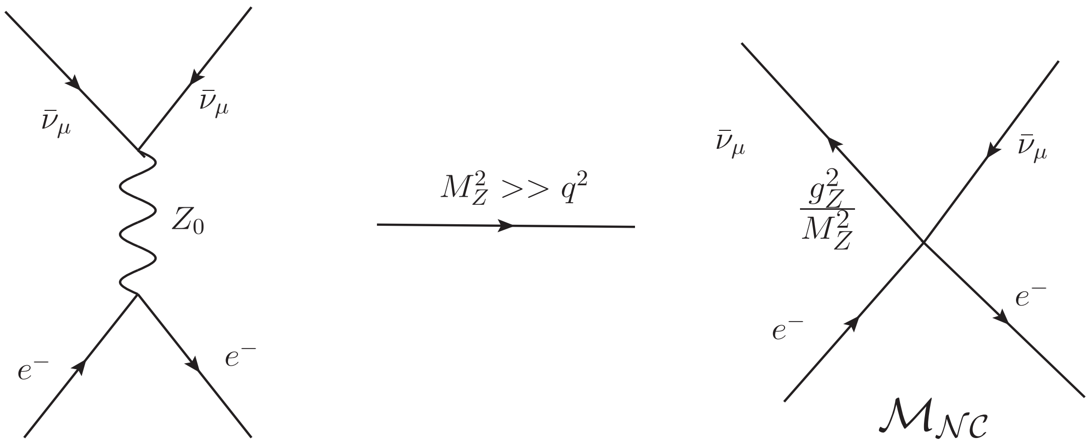
$\mathcal{H}_{eff}^{CC} = \frac{g_W^2}{M_W^2} J_\mu^{W+} J^{W-,\mu}$.

$= \frac{G_F}{\sqrt{2}} J_\mu^{W+} J^{W-,\mu}$

with $J_\mu^{W+} = \bar{\nu}^\mu (1-\gamma_5)\mu$ etc.

$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} = \frac{g_2^2}{8M_W^2}$

The Z exchange generates effective interaction Hamiltonian for neutral current processes such as : $e^- + \bar{\nu}_\mu \to \bar{\nu}_\mu + e^-$ OR $e^- + \nu_\mu \to \nu_\mu + e^-$.



$$g_Z = \frac{g_2}{\cos\theta_w}.$$

$$\mathcal{H}_{eff}^{NC} = \frac{g_Z^2}{2M_Z^2} J_\mu^Z J^{\mu,Z}$$

$$= \rho \frac{G_F}{\sqrt{2}} J_\mu^Z J^{\mu,Z}.$$

with $J_\mu^Z = g_L^f \bar{f}\gamma_\mu f_L + g_R^f \bar{f}\gamma_\mu f_R$

where

$$\rho = \frac{g_Z^2}{2g_W^2}\frac{M_W^2}{M_Z^2} = \frac{M_W^2}{M_Z^2\cos^2\theta_W}$$

$$\rho = \frac{g_Z^2}{2g_W^2}\frac{M_W^2}{M_Z^2} = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{\mathcal{G}_{NC}}{\mathcal{G}_{CC}}$$

ρ measures the ratio of strengths of the coupling in $\mathcal{M}_{\mathcal{NC}}$ and $\mathcal{M}_{\mathcal{CC}}$.

For the WS model $\rho = 1$ and Only if we choose doublet for the Higgs representation.

Initially measurements of neutral current processes gave $\rho \simeq 1$ and indicated correctness of this picture! Before the experimental discovery of the W/Z.

The couplings with the h depend on the representation to which Φ belongs. For the doublet,

$$\mathcal{L}_\Phi|_{unitarygauge} = \mathcal{L}_h + \left[M_W^2 W_\mu^+ W^{-\mu} + 1/2 M_Z^2 Z^\mu Z_\mu\right]\left(1 + \frac{h}{v}\right)^2$$

Not just the mass but the coupling of a Higgs to a gauge boson pair also comes from the Covariant Derivative term.

$g_{hVV} = \frac{M_V^2}{v}(-2i)$.

Thus hVV coupling is proportional to the gauge boson masses!

What about couplings of fermions to h?

It was really Weinberg's genius that he saw that exactly the same mechanism can be used to give masses to the fermions by postulating a gauge invariant term for interaction between the fermionic matter fields and the Higgs field!

$$\mathcal{L}^{e}_{yukawa} = -f^{*e}\bar{\mathcal{L}}_{1L}\Phi e_{1R} + h.c.$$

with $e_{1R} = e_R$ and $\bar{\mathcal{L}}_{1L} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$

Check this by working out Y and T_L for this Lagrangian. It should be a singlet under $SU(2)_L$ and $U(1)_Y$.

In the unitary gauge then with the choice of

$$\Phi' = \begin{pmatrix} 0 \\ 1/\sqrt{2}(v + h(x)) \end{pmatrix}$$

we get

$$\mathcal{L}^e_{yukawa} = -\frac{f^{*e}v}{\sqrt{2}}(\bar{e}_L e_R)(1 + h/v) + h.c.$$

Thus we have $m_e = +f^{*e}v/\sqrt{2}$ and the hee coupling is proportional to m_e.

Thus the leptons now have a nonzero mass. it is generated from a term which is gauge invariant. Just like the vector boson masses this mass issue too is sorted out in a gauge invariant way

The couplings of a fermion to the h is then proportional to m_e

Some extra work is needed for the case of quarks:

$$\mathcal{L}_{yukawa} = -f_{ij}^{*d}\bar{\mathcal{Q}}'_{iL}\Phi d'_{jR} - f_{ij}^{*u}\bar{\mathcal{Q}}'_{iL}\tilde{\Phi} u'_{jR} + h.c.$$

where $\tilde{\Phi} = i\sigma_2\Phi^*$.

Note : Both the terms are $SU(2)_L$ invariant (by construction). What abut $U(1)_Y$? Recall the gauge transformation $exp(-i\alpha_Y \frac{Y}{2})$.

One term involves Φ and the other $\tilde{\Phi}$ because we want the $\mathcal{L}$ to be invariant under $U(1)_Y$ also.

With, $Y_{\mathcal{Q}_L} = +1/3$, $Y_{d_R} = -2/3$, $Y_{uR} = +4/3$ the first term has $Y = 0$ with Φ, the second term has $Y = 0$ only with an $SU(2)_L$ doublet which has $Y = -1$. $\tilde{\Phi}$ most economic choice!

In the unitary gauge then with the choice of

$$\Phi' = \begin{pmatrix} 0 \\ 1/\sqrt{2}(v + h(x)) \end{pmatrix}$$

we then get

$$\mathcal{L}_{yukawa} = -\frac{f^{*d}_{ij}}{\sqrt{2}} v \; \bar{d}'_{iL}(1 + h/v) d'_{jR} - \frac{f^{*u}_{ij}}{\sqrt{2}} v \; \bar{u}'_{iL}(1 + h/v) u'_{jR} + h.c.$$

The mass matrices are:

$m^d_{ij} = +\frac{f^{*d}_{ij}}{\sqrt{2}} v$ and $m^d_{ij} = +\frac{f^{*d}_{ij}}{\sqrt{2}} v$

In general f^{*d}_{ij}, f^{*u}_{ij} are completely arbitrary matrices in the generation space.

The mass matrices are not diagonal in the basis d'_i, u'_i, in the most general case.

The states $d'_i, u'_i, i = 1 - 3$ are clearly not mass eigenstates.

So this means that mass eigenstates $d_i, u_i, i = 1, 3$ are linear combinations of d'_i, u'_i.

Recall that interaction is always given by the covariant derivative term.

$$\mathcal{L}_f = i\bar{\mathcal{Q}}'_{iL} D_\mu \gamma^\mu \mathcal{Q}'_{iL} + i\bar{u}'_{iR} D_\mu \gamma^\mu u'_{iR} + ...$$

For example $D_\mu \mathcal{Q}'_{iL} = \left[\partial_\mu - i\frac{g_2}{2}\vec{\sigma}\cdot\vec{W}_\mu - i\frac{g_1}{6}B_\mu\right]\mathcal{Q}'_{iL}$

The charged current interaction term written using the covariant derivative is for example:

$$\mathcal{L}^{cc} = \sqrt{2} g_2 \bar{u}'_{iL} \gamma^\mu d'_{iL} W^+_\mu + h.c$$

One can prove that in the most general case, after diagonalisation of both the m^d, m^u matrices this becomes

$$\mathcal{L}^{CC} = \sqrt{2} g_2 \bar{u}_{iL} \gamma^\mu d_{jL} \times \mathbf{V}_{ij}^{CKM} + h.c.$$

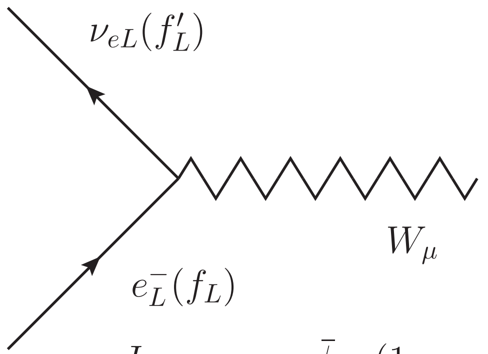
$\mathbf{V}^{CKM}$ is the Cabibbo-Kobayashi-Masakawa mixing matrix.

In your flavor physics course you will hear much more about it!

In fact for two generation case this is just the Cabibbo mixing matrix.

Step back to pre gauge theory days:

V-A theory had told that the basic unit of charged current weak interactions is a doublet of left handed fermions.



$J_\mu = g_{weak}\ \bar{\psi}\gamma_\mu(1-\gamma_5)\psi'$

$\Delta S = \Delta Q$ if $f = s, f' = u$

f and f' differ in charge by one unit. For strange quark case $\Delta S = 1$

Called Charged Current J_μ^{CC}.

Cabbibo's important observation Phys. rev. Lett. 10, 531 (1963)

The strength of all the three types of weak transitions $\Delta S = 0, \Delta S = 1$ and pure leptonic was equal *iff* appropriate doublet was

$$\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} \equiv \begin{pmatrix} u \\ d' \end{pmatrix}$$

θ_c called Cabbibo angle, experimentally determined value : $\sim 12^\circ$.

Interaction eignestate: $d' = d\cos\theta_c + s\sin\theta_c$. Bjorken and Glashow (1964) postulated a new quark c, with same quantum numbers as the u quark, which forms a doublet with orthogonal combination $s' = s\cos\theta_c - d\sin\theta_c$. The correct charged current coupling to W^+ is then $\bar{u}\gamma_\mu(1-\gamma_5)(d\cos\theta_c + s\cos\theta_c) = \bar{u}\gamma_\mu(1-\gamma_5)d'$.

Thus phenomenologically generation mixing was observed even before the Weinberg model was postulated.

The mass generation mechanism for quarks seems to naturally accommodate the quark mixing!

If $\mathbf{V}^{CKM}$ has to accommodate CP violation then it needs to be complex matrix. This needs three generations.

So this was the prediction of of Kobayashi and Maskawa that if the SM has to 'describe' the observed CP violation in terms of quark mixing one needs three generations!

We already noticed that in the SM the way we have constructed it there are NO generation changing couplings of the Z boson

Recall $J^Z_\mu = g^f_L \bar{f}\gamma_\mu f_L + g^f_R \bar{f}\gamma_\mu f_R$

Now you can check easily for example, $\bar{d}\gamma_\mu d_L + \bar{s}\gamma_\mu s_L = \bar{d}'\gamma_\mu d'_L + \bar{s}'\gamma_\mu s'_L$

Recall also g^f_L and g^f_R are the same for d, s. Hence J^Z_μ is the same whether expressed in terms of the interaction eigenstates or the mass eigenstates.

This happens because the left handed quark fields in the successive generations belong to exactly the same representation of the EW gauge group.

This was a theorem proved by Glashow and Weinberg.

The next point we wish to discuss is how the SM 'naturally' accommodates the observed suppression of the flavour changing neutral current processes: FCNC at loop level as well. Before we engage with that, let us discuss a bit more about Cabibbo mixing again.

Current eigenstates that couple to the W are two doublets:

$$\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} \qquad \begin{pmatrix} c \\ -d\sin\theta_c + s\cos\theta_c \end{pmatrix}$$

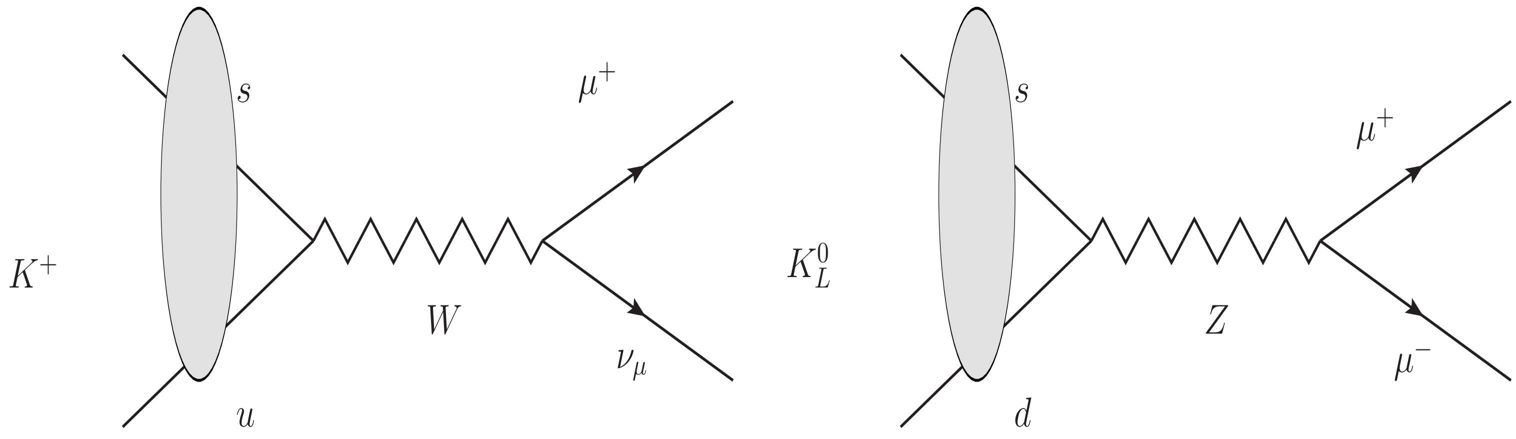
Quark mixing in two generations can then be represented by

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$J_\mu^{CC} = \bar{d}'\gamma_\mu(1-\gamma_5)u + \bar{s}'\gamma_\mu(1-\gamma_5)c$$

(recall the $f\bar{f}'W$ vertices we saw yesterday)

Charm was postulated. Existence of the charm does the trick of making Flavor Changing Neutral Current vanish at least at tree level by making sure that a vertex $d\bar{s}Z$ does not exist. What would have gone wrong if it had existed?



Weak CC *Weak NC*

K^+, $\bar{s}u$ bound state. $K^+ \to \mu^+\nu_\mu$: weak charged current decay, $\Delta S = 1$. Happens at usual weak rates.

K^0 is a $\bar{s}d$ bound state. If a weak neutral current with $\Delta S \neq 0$ (Flavor Changing Neutral Current: FCNC) were to exist with the same strength as the weak charged current. It does not happen in practice.

Because $K_L^0 \to \mu^+\mu^-$ happens very rarely (one part in 10^8 among all K_L decays)

Once we have two quark doublets, tree level FCNC vanishes automatically. ☺

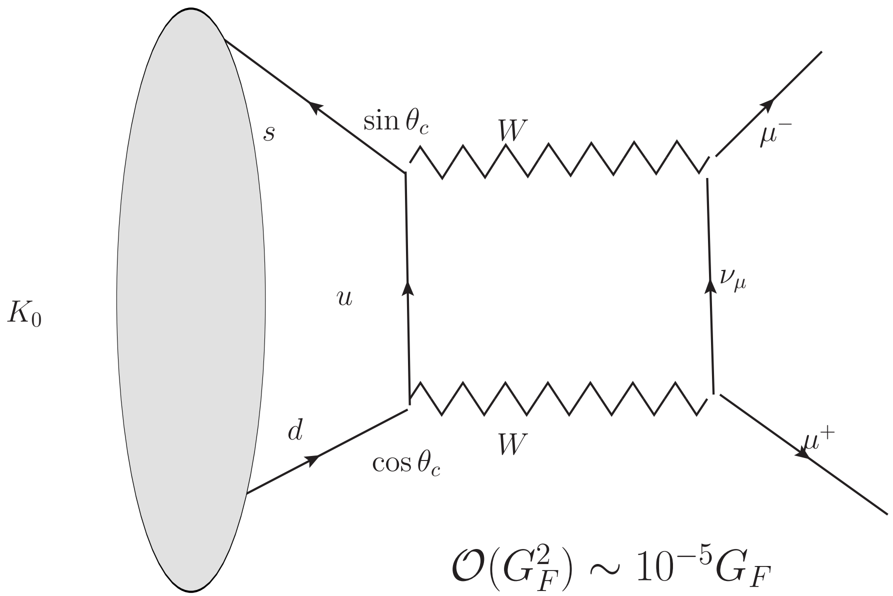
Is Rabi's question answered? We somehow show that we need a new quark.

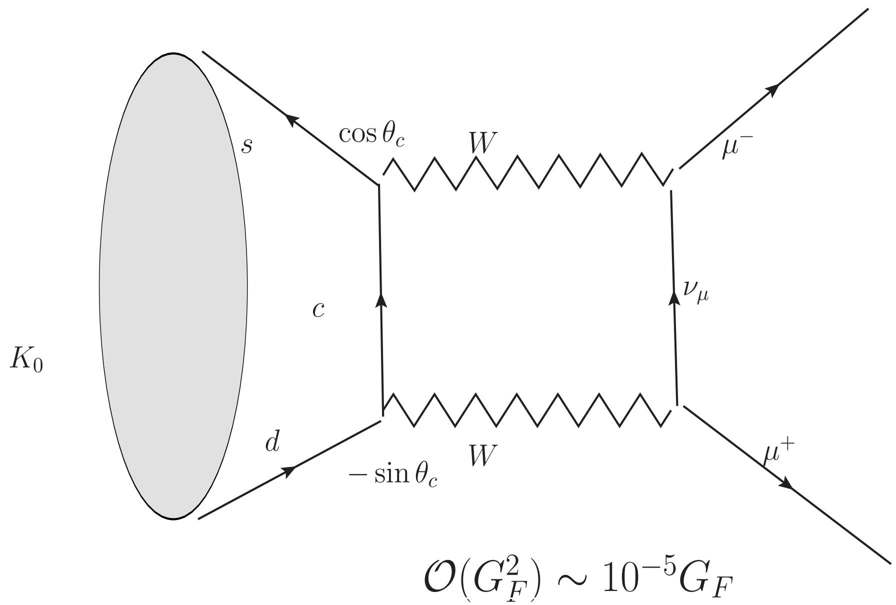
Still no answer to Rabi's question 'who ordered the μ the second generation of leptons?

Bute can we say FCNC ordered a new quark?

What this tells is that all the left handed fermions have to appear as SM doublets.

Second point : Quantum properties of the $SU(2)_L \times U(1)$ gauge theory imply that the number of generations should be equal for quarks and leptons. (Anomaly cancellation!) (Will mention in the end if possible)





What happens to FCNC with loops?

If charm contribution in the second diagram is not included the prediction for this flavour changing decay will be much too big compared to data.

<u>Absence of Flavour Changing Neutral Currents is granted in the EW</u>

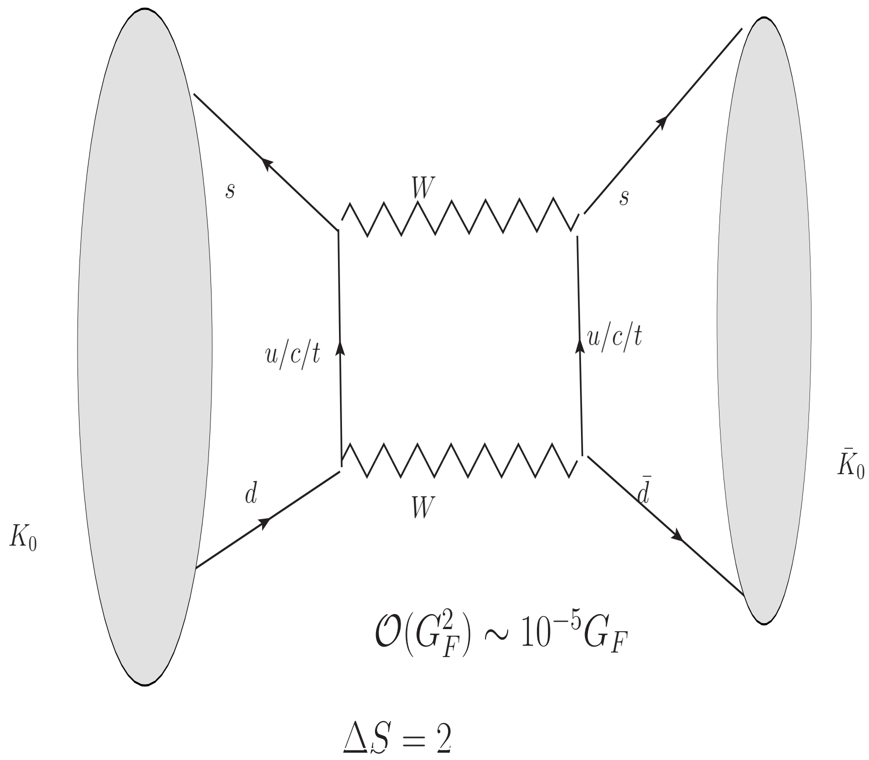
theory ONLY IF CHARM exists. FCNC prediction will be exactly zero if $m_c = m_u$.

Of course in real life $m_c \neq m_u$. So the FCNC exists but depends on $m_c^2 - m_u^2$. Observed FCNC can then constrain m_c

For any physics beyond SM this is always a constraint that HAS to be satisfied.

This cancellation is an example of the Glashow-Iliopoulos-Maiani (GIM) cancellation mechanism.

A very simplistic presentation given here.



Loop yields a finite result only in the four quark picture the result of the calculation and is

$$\frac{\Delta M_K}{M_K} = \frac{G_F^2}{4\pi} m_c^2 \cos^2\theta_c \sin^2\theta_c f_K^2$$

$= 7 \times 10^{-15}$

Predicts $m_c \sim 1.6$ GeV.

The November revolution: Discovery of Charmonium at 3.1 GeV was the first step in validation of the SM as a renormalisable Gauge theory!

NEEDS GIM cancellation!

Value of $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_w}$ depends on the representation of the Higgs.

Consider a real $SU(2)_L$ triplet $(T = 1, Y = 0)$, scalar : $\Xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$

Here, $Q = T_{3L} + \frac{Y}{2}$. Only ξ^0 can develop nonzero vev. We then have $\langle \Xi \rangle_0 = \begin{pmatrix} 0 \\ v_{3r} \\ 0 \end{pmatrix}$.

Calculate again $(D_\mu \xi)^\dagger (D^\mu \xi)$. We find Z now remains massless. Thus the desired symmetry breaking is not brought about by this scalar field. Only three scalar fields and one field develops the nonzero vev. We dont have enough Goldstone degrees of freedom to give mass to all the gauge bosons.

On the other hand if one uses a complex $SU(2)_L$ triplet with $T = 1, Y = 2$, then

$$\widetilde{X} = \begin{pmatrix} \chi^{++} \\ \chi^{+} \\ \chi^{0} \end{pmatrix}$$

Only $\mathcal{R}e\chi^0$ can develop nonzero vev.

$$\langle\widetilde{X}\rangle_0 = \begin{pmatrix} 0 \\ 0 \\ v_{3c} \end{pmatrix}.$$

Now we have 6 scalar fields and enough Goldstone modes are available to give masses to both W and Z. Calulating $(D_\mu\widetilde{X})^\dagger(D^\mu\widetilde{X})$ gives us $\rho = 2$. The model also has two doubly charged Higgses.

For a Higgs doublet we can trace $\rho = 1$ to an accidental symmetry of the Higgs potential.

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\Phi^\dagger \Phi = \Re e(\phi_1)^2 + \Im m(\phi_1)^2 + \Re e(\phi_2)^2 + \Im m(\phi_2)^2$$

Has an $SO(4)$ symmetry for the rotation of a vector

$$X = \begin{pmatrix} \Re e(\phi_2) \\ \Im m(\phi_2) \\ \Re e(\phi_1) \\ \Im m(\phi_1) \end{pmatrix}$$

Of course after the SSB this symmetry is lost as ONLY one field gets nonzero vev.

However an $SO(3)$ symmetry is still retained as the remaining all three have a zero vev. This is reflected in the equality of mass term for all W^a_μ. This guarantees that $\rho = 1$

Remember the discussion of last lecture of Glashow paper. In that matrix if one puts $M^2_{W^a} = M^2_W$, one gets $\rho = 1$.

Thus whatever may be the mass generation mechanism as long as one respects this symmetry $\rho = 1$. However, this symmetry is broken, e.g., from mass splitting between the two members of $T_L = 1/2$ doublet of $SU(2)_L$. In the limit of these being small, ρ will remain close to one in the SM even after radiative corrections! Will come to this in the last lecture.

Of course if we find evidence of ρ differing from the value predicted even after including radiative corrections then we start looking for contributions from additional Higgs multiplets! Can be relevant for possible M_W discrepancy.

Determination of Neutral Current couplings and hence $\sin^2\theta_w$ (circa 1981). Also gave $\rho \sim 1$.

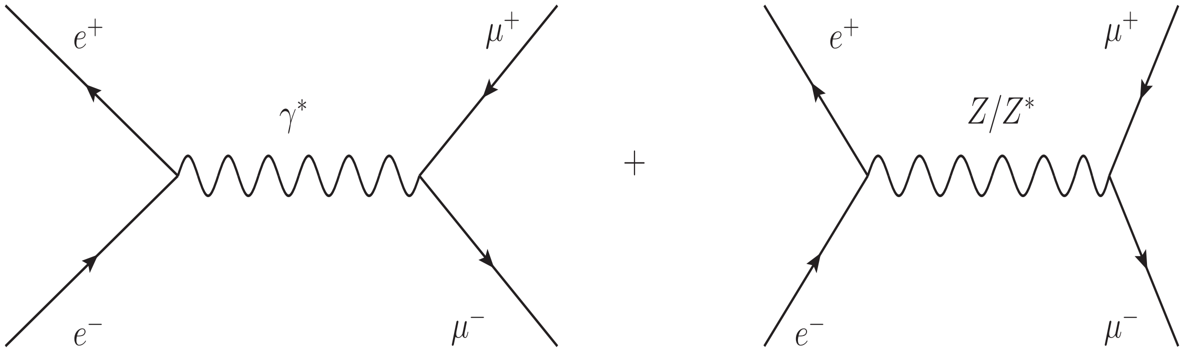
So how were the weak neutral current couplings were measured? Recall they were all given in terms of just $\sin\theta_w$.

This is independent of M_Z. At low energies analyse the processes in the language of contact interaction. At high energies include the Z mass.

f	ν	e^-	u	d
g_L^f	$\frac{1}{2}$	$-\frac{1}{2}+\sin^2\theta_W$	$\frac{1}{2}-\frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W$
g_R^f	0	$\sin^2\theta_W$	$-\frac{2}{3}\sin^2\theta_W$	$\frac{1}{3}\sin^2\theta_W$
g_A^f	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
g_V^f	$\frac{1}{2}$	$-\frac{1}{2}+2\sin^2\theta_W$	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$

Process	$d\sigma/dy$	σ
$\nu_\mu + e^- \to \mu^- + \nu_e$	$A\ s(g_L^\nu)^2(g_L^e)^2$	$A\ s\ (g_L^\nu)^2(g_L^e)^2$
$\nu_\mu + e^- \to \nu_\mu + e^-$	$A\ s(g_L^\nu)^2\left[(g_L^e)^2 + (1-y)^2(g_R^e)^2\right]$	$A\ s\ (g_L^\nu)^2[(g_L^e)^2 + \frac{1}{3}(g_R^e)^2]$
$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$	$A\ s\ (g_L^\nu)^2\left[(g_R^e)^2 + (1-y)^2(g_L^e)^2\right]$	$A\ s\ (g_L^\nu)^2\left[\frac{1}{3}(g_L^e)^2 + (g_R^e)^2\right]$
$\nu_e + e^- \to \nu_e + e^-$	$A\ s\ (g_L^\nu)^2\left[(g_L^e+1)^2 + (1-y)^2(g_R^e)^2\right]$	$A\ s\ (g_L^\nu)^2\left[\frac{1}{3}(g_R^e)^2 + (g_L^e+1)^2\right]$
$\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$	$A\ s\ (g_L^\nu)^2\left[(g_R^e)^2 + (1-y)^2(g_L^e+1)^2\right]$	$A\ s\ (g_L^\nu)^2\left[\frac{1}{3}(g_L^e+1)^2 + (g_R^e)^2\right]$

In addition $e^+e^- \to \mu^-\mu^+$ process involves the same neutral current couplings.



γ interactions are parity conserving and Z interactions are parity violating.

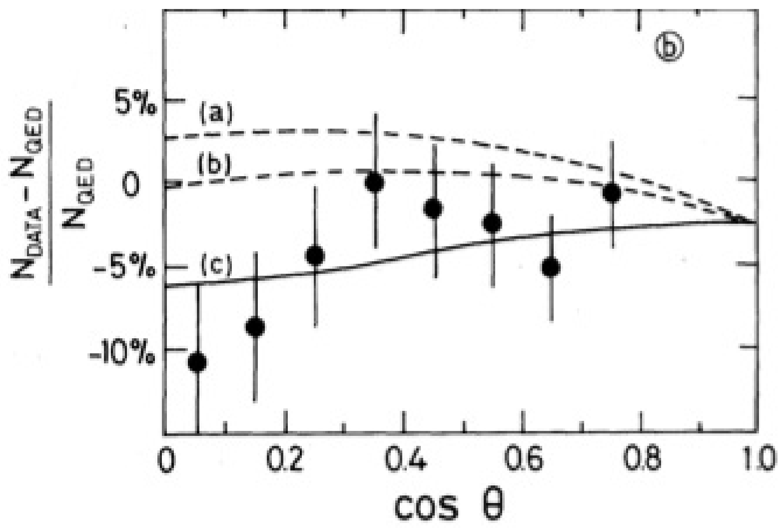
Hence the interference as well as pure Z contribution will cause asymmetric emission of μ^- wrt the (say) e^- emission. A forward-backward asymmetry

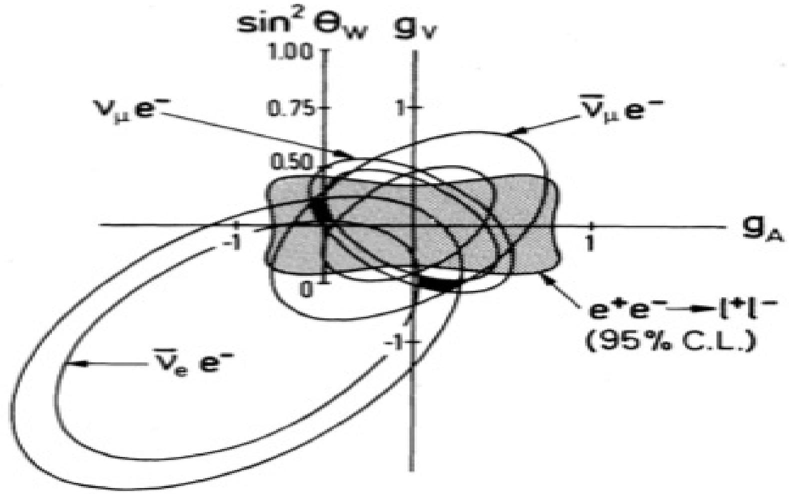
One can calculate the FB asymmetry

$$A^{\mu}_{FB}| = \frac{\sigma(\cos\theta^{\mu} > 0) - \sigma(\cos\theta^{\mu} < 0)}{\sigma(\cos\theta^{\mu} > 0) + \sigma(\cos\theta^{\mu} < 0)}$$

$$A^{\mu}_{FB}|_{s \ll M_Z^2} = -\frac{3}{\sqrt{2}}\frac{G_{\mu}s}{e^2}g_A^2\frac{1}{1-\frac{4G_{\mu}s}{\sqrt{2}e^2}g_V^2}; \qquad A^{\mu}_{FB}|_{s=M_Z^2} \sim \frac{g_A^2 g_V^2}{\left(g_A^2+g_V^2\right)^2}.$$

where s is the square of the c.m. energy. In the 80's one had PEP/PETRA going upto $\sqrt{s} = 30$ GeV and the TRISTAN upto $\sqrt{s} = 60$ GeV. Universality of all the lepton couplings with the Z is assumed here.





$\sin^2\theta_W = 0.27 \pm 0.08$. Validated the EW unification idea. This gave GSW their Nobel Prize already in 1979 but the measurement of $\sin^2\theta_W$ was poor and precision for predicted M_W, M_Z was $\sim$ 10 GeV.

Note our energies were way below the M_W, M_Z. 'indirect' effects can give us information about the physics at much higher scales.

This is the reason for pushing for precision calculations and precision measurements.

A better measurement of $\sin\theta_w$ came from νN scattering experiments and polarised e^--Deuterium scattering experiments, assuming the SM: $\sin^2\theta_w = 0.229 \pm 0.009$ (One assumed doublet Higgs)

This gave a SM prediction (indirect) for the masses :

$M_W \simeq 78.15 \pm 1.5$ GeV; $M_Z \simeq 89 \pm 1.3$ GeV.

The UA-1/UA-2 measurement was

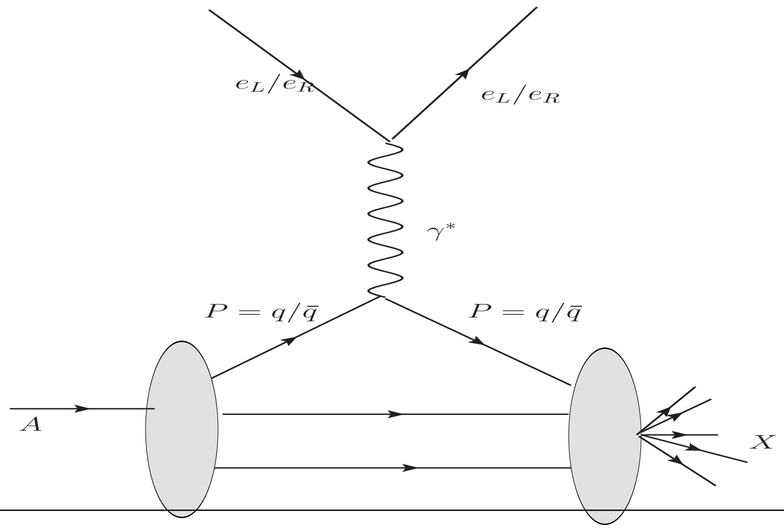
$M_W = 80 + 10 - 6$ GeV , $M_Z = 91.9 \pm 1.3 \pm 1.4$ GeV.

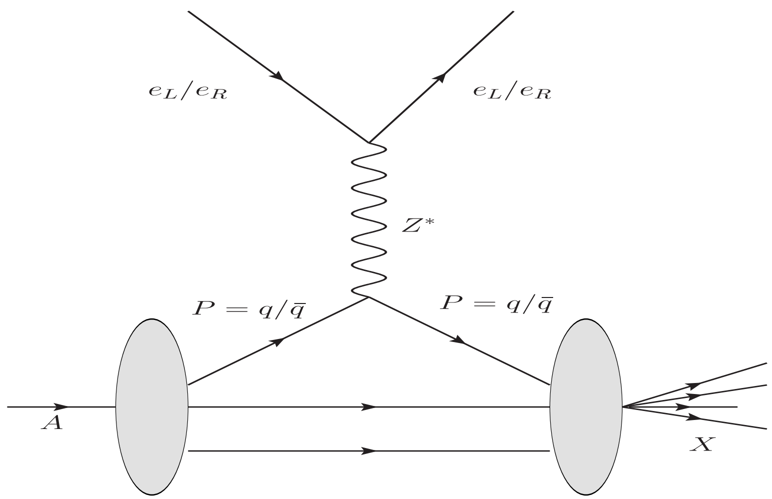
SM prediction agreed within errors with the **measured** values. (Rubbia and Van der Meer got their Nobel prize for this).

1) Evidence of right handed charged currents!

2)Disagreement with the SM prediction of polarisation asymmetry of DIS cross-section.

But both vanished soon (as Mark Twain said the rumours of the death were highly exagirated). The DIS gave one of the most accurate determination of $\sin^2\theta_w$ at that time, because the effect was linearly dependent on g_V^e.



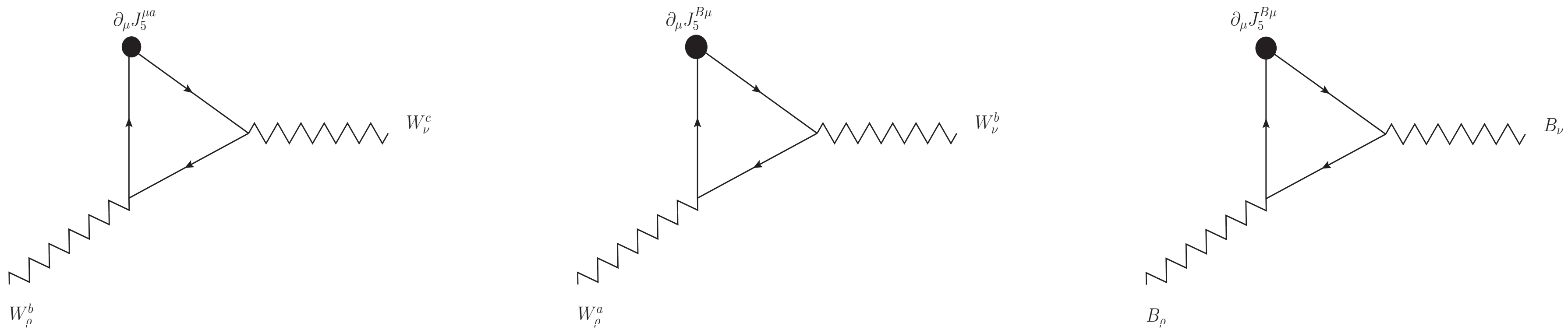


I already mentioned constraints implied by anomaly cancellations. Anomalies direct us to the correct particle content of the theory!

What are anomalies? Gauge invariance of axial current $J_\mu^5 = \bar{\psi}\gamma_\mu\gamma_5\psi'$ is not preserved in a quantum theory in not preserved by dimensional regularisation due to the presence of γ_5. Ie. even if we start with $\partial_\mu J_5^\mu = 0$ at loop level the RHS develops a nonzero value.

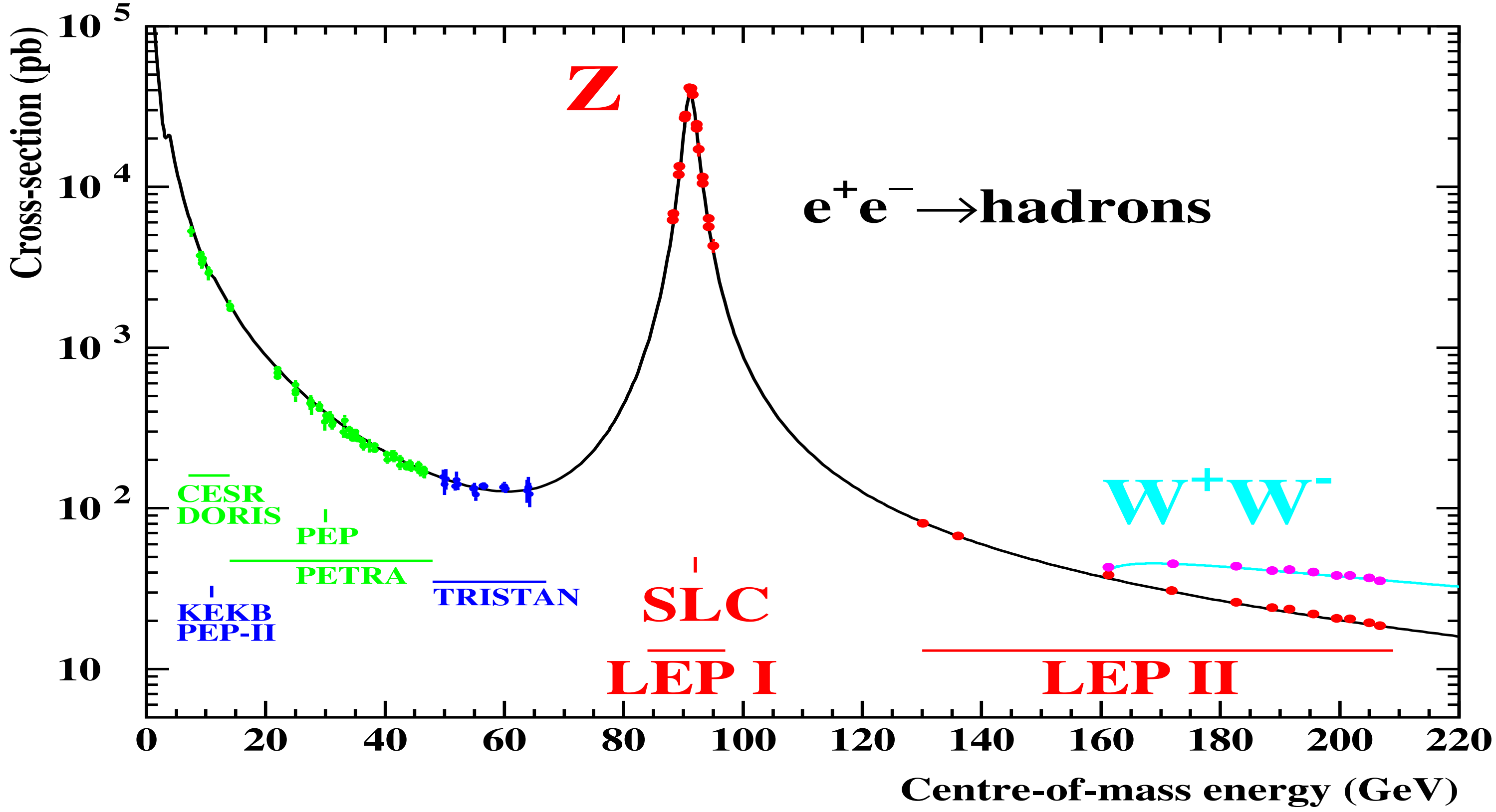
For renormalisbility of the theory gauge invariance has to be preserved. But such nonzero value will break it and hence spoil the renormalisability.

Following triangle diagrams can give nonzero contribution to the $\partial_\mu J_5^\mu$.



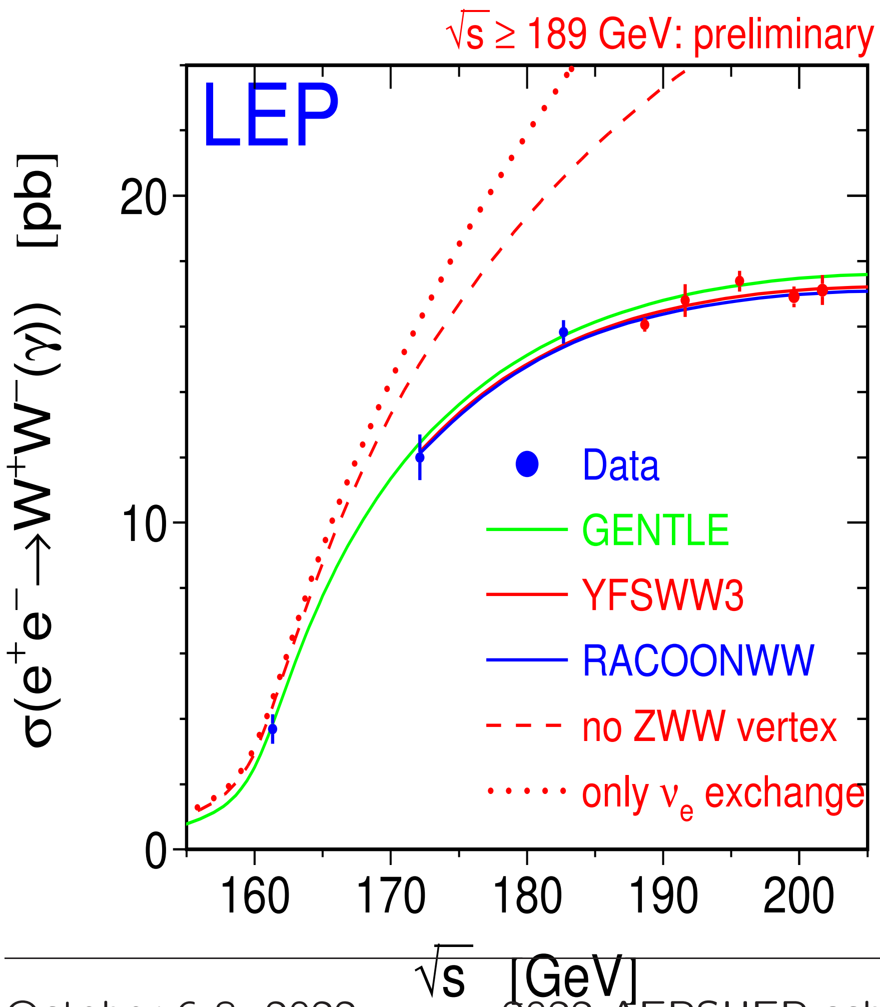
Luckily these contributions, are independent of the mass of the fermion in the loop except for a coefficient decided by the representation to which the fermion belongs. Hence way to handle this is if the sum over all fermions in the loop vanishes. Anomaly cancellation

The requirement then predicts that SM must have equal number of quark and lepton generations.



Solid line is the SM fit. Phys. Rept. 427, 257 (2006). Large electromagnetic and QCD radiative corrections. Initial state radiation makes the curve asymmetric near the resonance.

Direct 'Proof' of Symmetry and Symmetry breaking!!



Proof that electroweak symmetry exists and that it is broken.

The triple gauge boson ZWW coupling tames the bad high energy behaviour of the cross-section caused by the t-channel diagram. Direct proof for the ZWW coupling.

This and precision testing, confirm basics of the SM