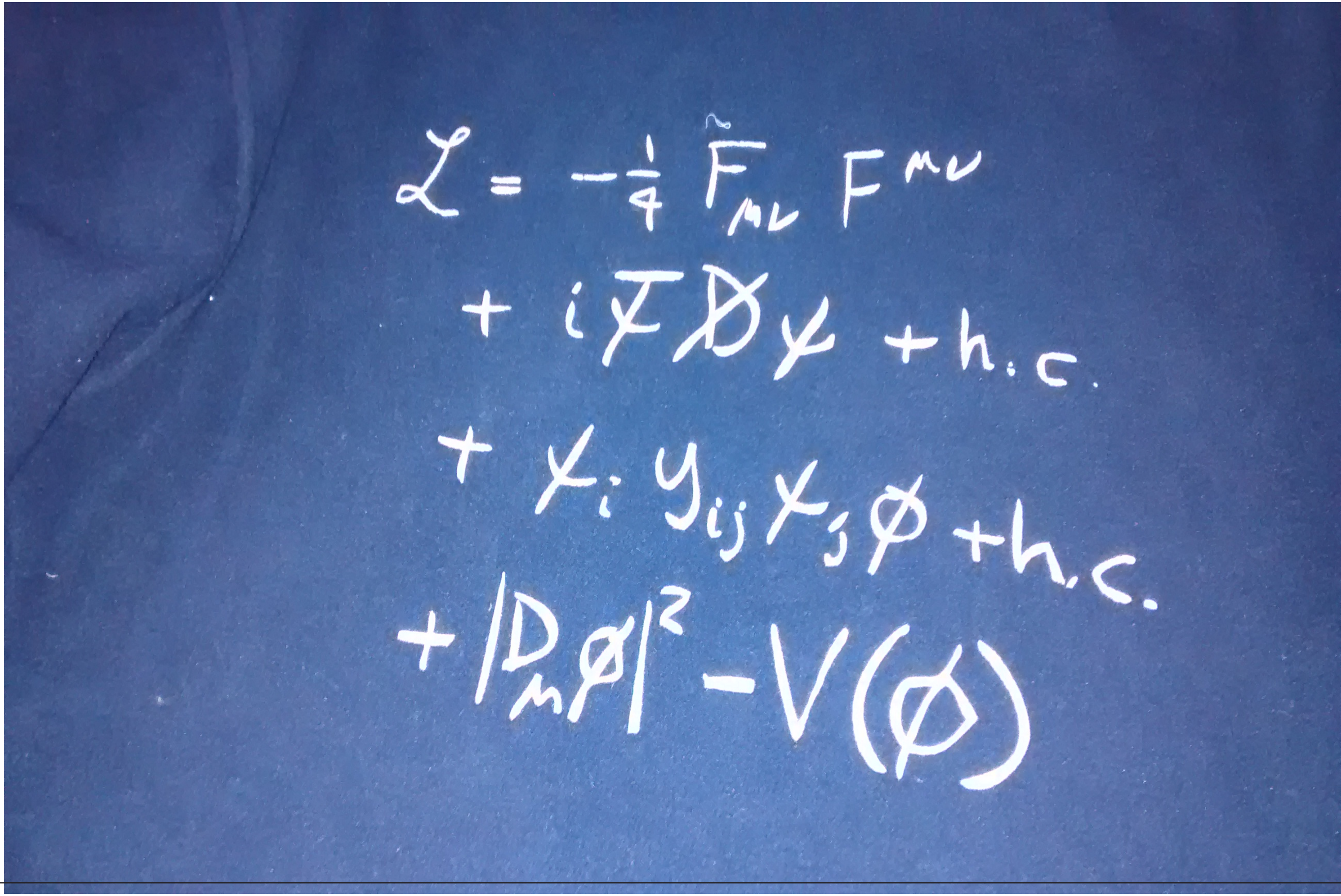
Field Theory and EW Standard Model-IV


$\mathcal{L} = -\frac{1}{4} \tilde{F}_{\mu\nu} F^{\mu\nu}$
$+ i\bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

Lectures 1 and part of Lecture 2:

Mainly Physics of the Weak Bosons:

a) Setting up the notation of the SM Lagrangian, including Higgs mechanism

b) How one can understand the development of the SM also in terms of taming the bad high energy behavior of the scattering amplitudes!

c) The miracles of the particle spectrum of the SM: Anomaly cancellation and the Custodial symmetry!

Lecture 2 :**Prediction of new particles and their masses in the SM. Fermions, their couplings to W/Z in the SM and testing the SM at tree level.**

a) Flavour mixing, CKM matrix

b) Flavour changing neutral current, GIM and all that

Prediction of M_c from the observed mass difference K_L–K_S. The 'first' use of an indirect effect to predict a mass!

c) Test of EW unification with the determination of $\sin\theta_w$ and resultant test of a unified gauge field theoretic description of Electro Weak interactions.

Still left: Precision testing of the tree level predictions!

Lecture 3: SM development: A story of theoretical predictions for existence and masses of new particles being confirmed by experiments and vice versa.

a) Radiative corrections in a spontaneously broken gauge theory, oblique corrections and precision testing of the SM.

b) 'Indirect' determination of the mass of the top and Higgs!

c) Theoretical bounds on the Higgs mass

d) Implications of the measured mass of the Higgs for the SM! i.e the scale unto which SM can be consistent without any additional physics!

May not be able to cover c and d in detail but enough to vett your appetite.

Let me begin by the summary of physics ideas we learnt so far.

1)What were the reasons which led people to believe weak interactions could be described as a gauge theory

2)The need of a 'partial symmetry' (Glashow's name) ie. the symmetry group for the EW unification not being a simple gauge group.

3)Need of SSB and the content of the SM with a doublet Higgs. Prediction of equal strengths of the neutral and charged current effective interactions.

4)SM predicts masses of all the bosons in terms of just two parameters $\sin\theta_w$ and λ. The masses of the W, Z depend on the Higgs representation used for the SSB.

5) Couplings of all the fermions to gauge bosons decided by the representation to which they belong and hence are 'universal' ie. same for all generations.

6) Masses of the fermions need arbitrary Yukawa couplings.

7) Enough freedom in the mass generation mechanism to allow for accommodating the experimentally observed intergenerational mixing in quark sector.

8) Intergenerational mixing in the charged current in the quark sector and absence thereof in neutral current can be avoided at the tree level by making sure representation structure is repeated across generations.

9)Observed suppression of FCNC led to postulation of c quark. Suppression beyoat loop level was explained by GIM in 4-quark picture, predicing m_c (B.W. Lee/Gaillard). The observation of J/ψ giving first 'proof' of gauge theory of weak interactions.

10) Necessity of three generations of quarks to accommodate observed CP violation in terms of quark mixing.

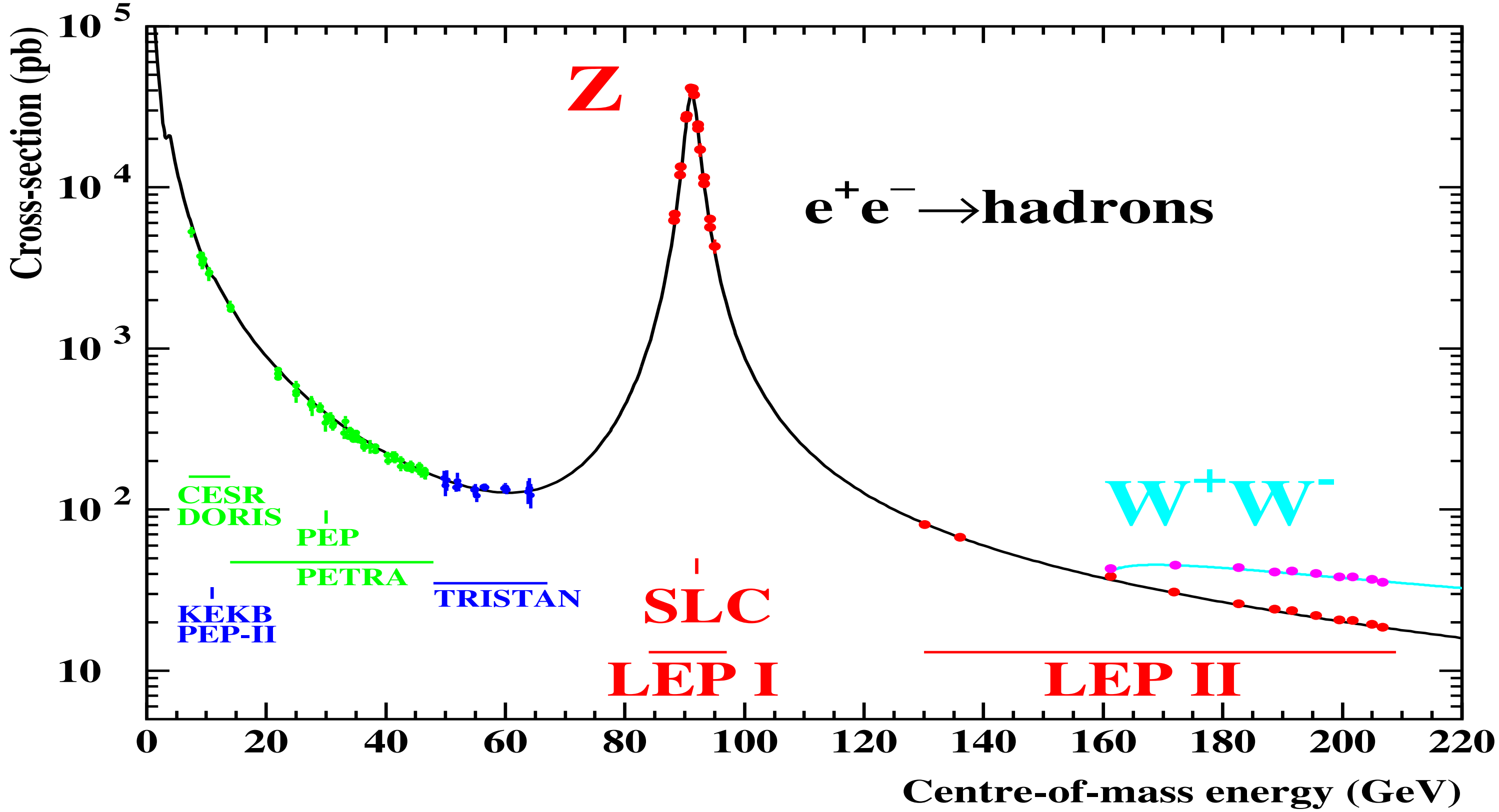
11)Existence of the same number of generations of leptons and quarks required by Anomaly cancellation.

12) The rate of Higgs production in gluon-fusion channel at the LHC rules out existence of a sequential fourth generation of chiral quarks which gets its mass via SSB

14)Lepton number and baryon number conservation accidental symmetries of SM.

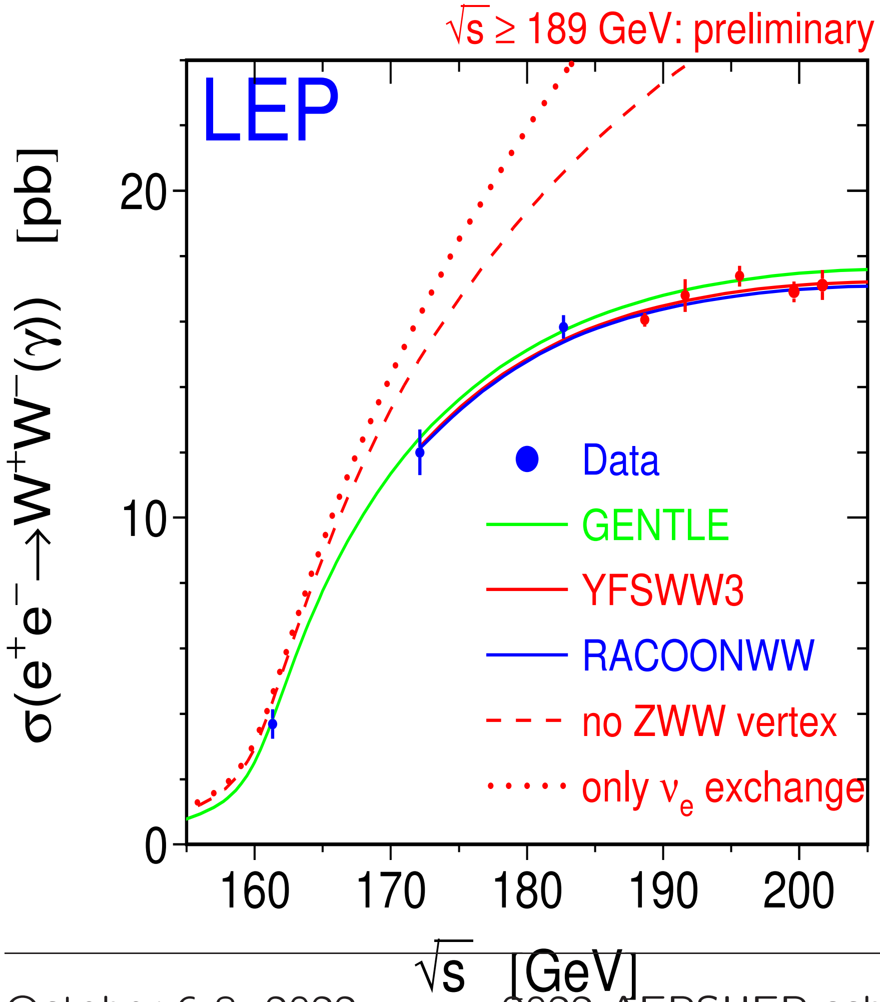
We did not discuss last two

After the early ’confirmation’ of ’EW unification’ high precision experiments followed.



Solid line is the SM fit. Phys. Rept. 427, 257 (2006). Large electromagnetic and QCD radiative corrections. Initial state radiation makes the curve asymmetric near the resonance.

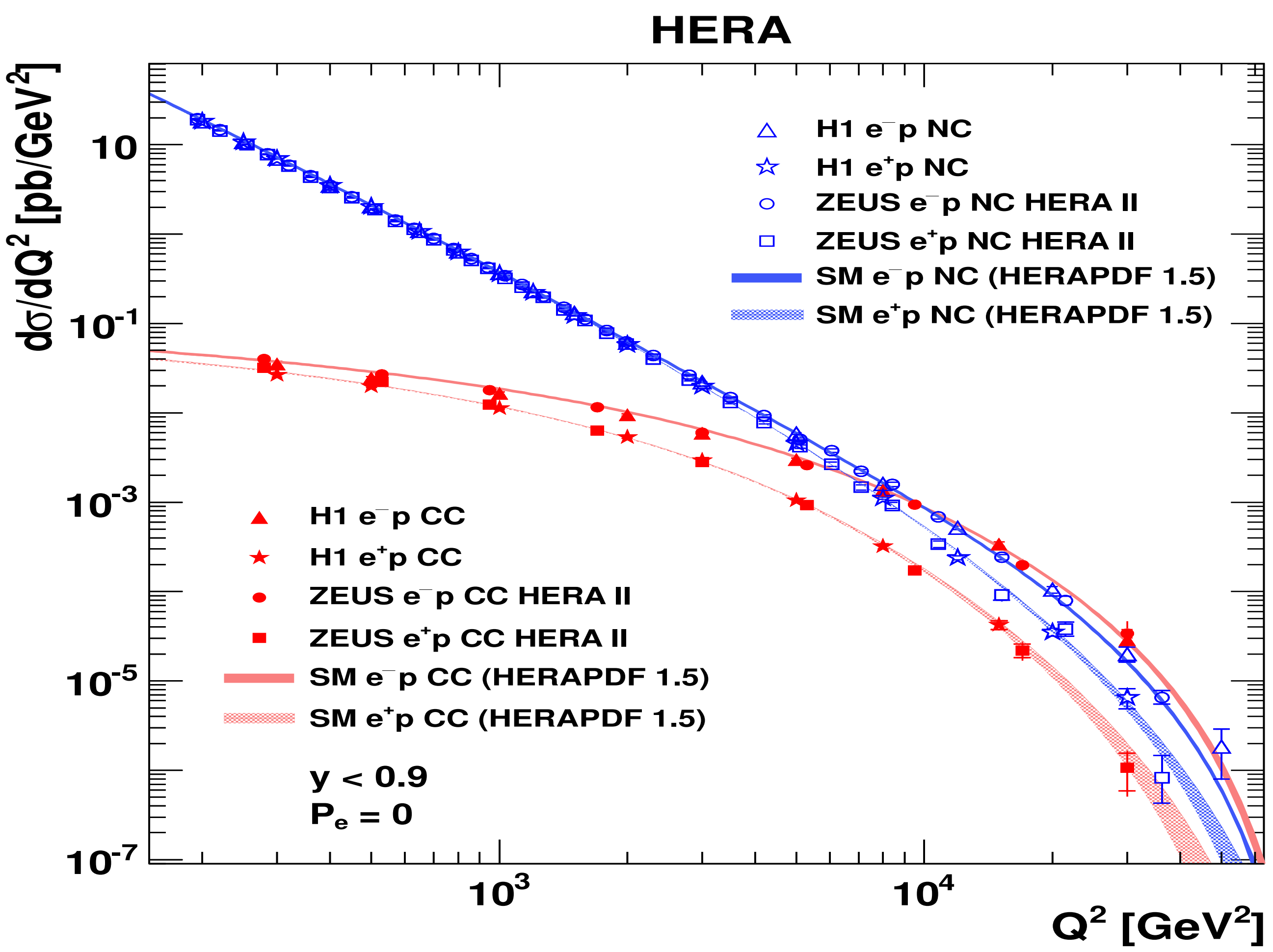
Direct 'Proof' of Symmetry and Symmetry breaking!!



Proof that electroweak symmetry exists and that it is broken.

The triple gauge boson ZWW coupling tames the bad high energy behaviour of the cross-section caused by the t-channel diagram. Direct proof for the ZWW coupling.

This and precision testing, confirm basics of the SM



Deep Inelastic Scattering:

NC : $e^- + p \to e^- + X$ (sum of γ and Z exchange).

CC : $e^- + p \to \nu_e + X$ (W exchange)

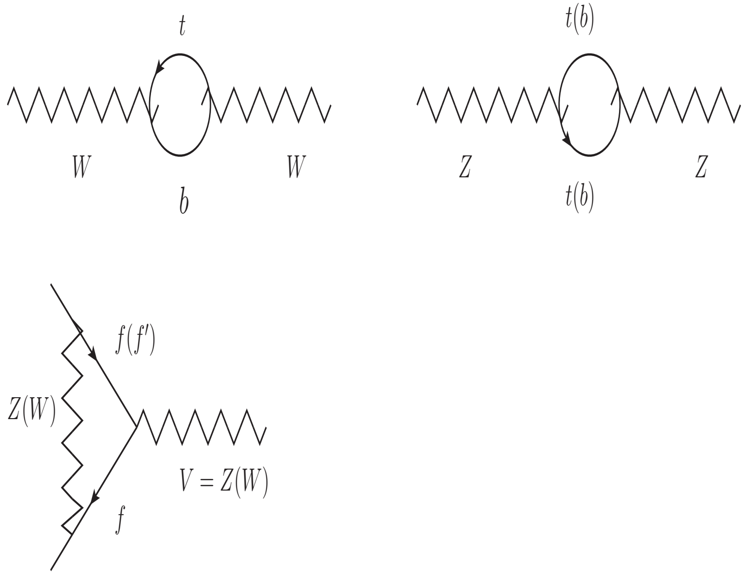
The NC and CC cross-sections merge around $Q^2 = (90GeV)^2$.

In making these comparisons one has used the tree level expressions for all the quantities.

eg. $M_W = \frac{g_2 v}{2}$

All these tree level relations change due to quantum corrections. Renormalisability guarantees that these corrections are finite! Renormalisability in turn guaranteed by gauge invariance and that is guaranteed by SSB/Higgs boson.

Order of the day:1)look for the Higgs 'directly' and 2)Precision test SM through measuring the loop effects on observables and properties of the W, Z.



$$\rho_{corr} = 1 + \Delta\rho$$

$$\Delta\rho \simeq \frac{3G_F M_t^2}{8\pi^2\sqrt{2}} = 0.01$$

There is also a diagram with h in the loop.

The corrections for the Z and W are different. The dominant corrections come from loop containing the heaviest quarks t, b (and sub dominant ones from h) ρ changes from value 1. (Veltman: screening theorem about the h contribution being small) Before top quark was found, its value was indirectly obtained from measuring ρ.

The corrections can be calculated only if theory is renormalisable.

Renormalisability proved by 't Hooft and Veltman endows calculability and predicitivity

Precision measurements at the LEP-I of Z properties and all the neutral current couplings as well as precision measurements of the properties of the W at LEP-200, tested these corrections!

Such a test at the loop level of the relations should indicate a a finite mass for the Higgs if the theory is indeed renormalisable and would be an an indirect proof for the Higgs!

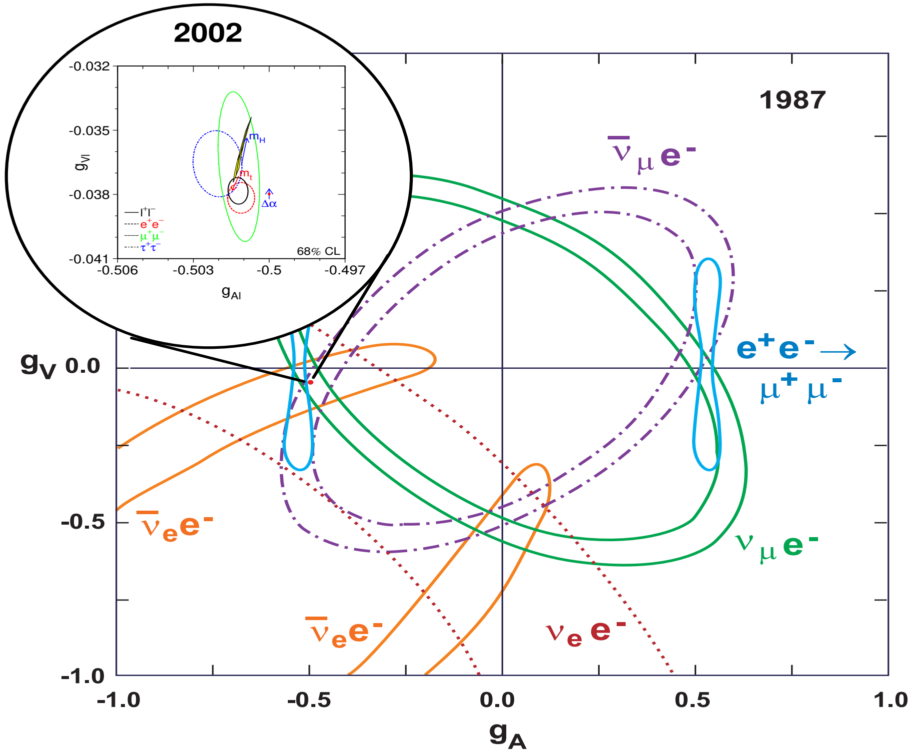
High precision measurements require high precision calculations.

Higher order QED and QCD corrections highly important and non-trivial.

Good understanding of QCD to calculate correctly what the detectors observe: jets.

Extensive collaborative studies between experimentalists and theorists LEP Yellow Reports.

These measurements tested the tree level couplings and more!



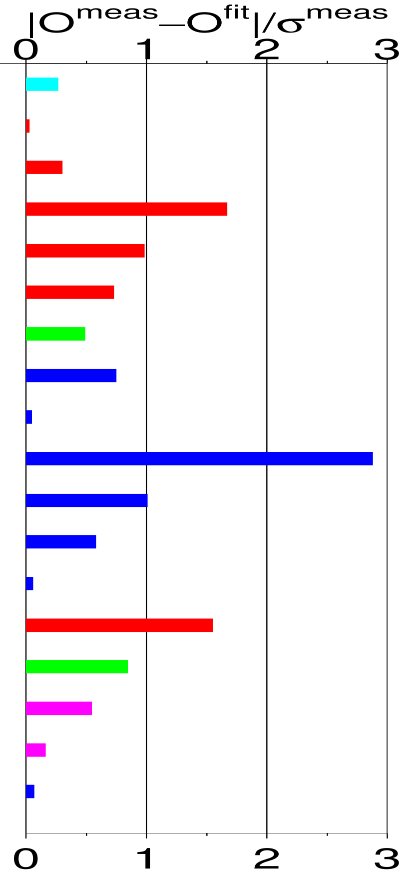
Enormously more precise measurements.

Agreement with SM prediction would have been impossible unless the predicted values included higher order corrections, calculated in perturbation theory.

Recall correction to $\Delta\rho$ is 1% . The measurement is accurate to 1 part in 100 or better to see confirm this. Large mass of the t made this effect measurable!

Analog of $(g-2)_\mu$ for QED!

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



March 2011

see http://lepewwg.web.cern.ch

Logical steps in Precision testing of the SM and the indirect limits:

• SM has three parameters g_2, g_1 and v. All the SM couplings, gauge boson masses functions of these.

• A large number of EW observables measured quite accurately.

• M_Z, α_{em} and G_F are most accurately measured. Trade g_2, g_1 and v for these.

- All observables depend on these three apart from M_f (mainly M_t) *and* M_h, and of course α_s.

- Calculate all observables using **1 loop EW** radiative corrections which can be computed in a renormalisable quantum field theory.

- Compare with data, make a SM fit. Tests the SM at loop level.

Given α_{em}, M_Z, G_μ one can calculate M_W using tree level relations.

$\alpha_{em} = 1/137.0359895(61)$, $G_\mu = 1.16637(1) \times 10^{-5} GeV^{-2}; MZ = 91.1875 \pm 0.0021$ GeV

Calculate M_W using the tree level relation

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{\pi\alpha}{2M_W^2(1-M_W^2/M_Z^2)}$$

$M_W^{tree} = 80.939$ GeV and $M_W^{expt} = 80.385 \pm 0.015$ GeV.

Loop corrections needed. Renormalisability guarantees that all the corrections are finite!

Loop corrections can be calculated consistently only in a renormalizable theory.

Depend on m_h logarithmically and on m_t quadratically.

Compare measured values of M_W, m_t against calculated from EWPT for different values of m_h.

Precision measurements and precision calculations!

LEP legacy, augmented by Tevatron precision measurements!

Now by the LHC. In future by ILC,FCC,CLIC..?

March 2011:

$M_W = 80.385 \pm 0.015$ GeV (measured), $80.377 GeV (theory)$

$m_t = 173.20 \pm 0.90$ GeV (measured) 172.26 GeV (theory)

In fact before top mass was measured at the Tevatron the fits made a prediction for it. The agreement between measurement and prediction was a triumph. Veltman and 't Hooft got the Nobel prize only after that!

The current values now are a little different. Tevatron, LHC have added to the precision. Once top was found and m_t measured the game was to predict m_h. The value of χ^2 at the minimum is not great! so some people were bothered by it pre July 2012!

Absence of FCNC $\Rightarrow$ quarks must come in isospin doublets, charm was predicted. Top was expected to be present once b was found

Prediction of M_c, M_t from flavour changing neutral current processes such as $K_0 - \bar{K}_0$ and $B_0 - \bar{B}_0$ mixing. Agreement with experimentally measured values 'proves' gauge theory of EW interactions.

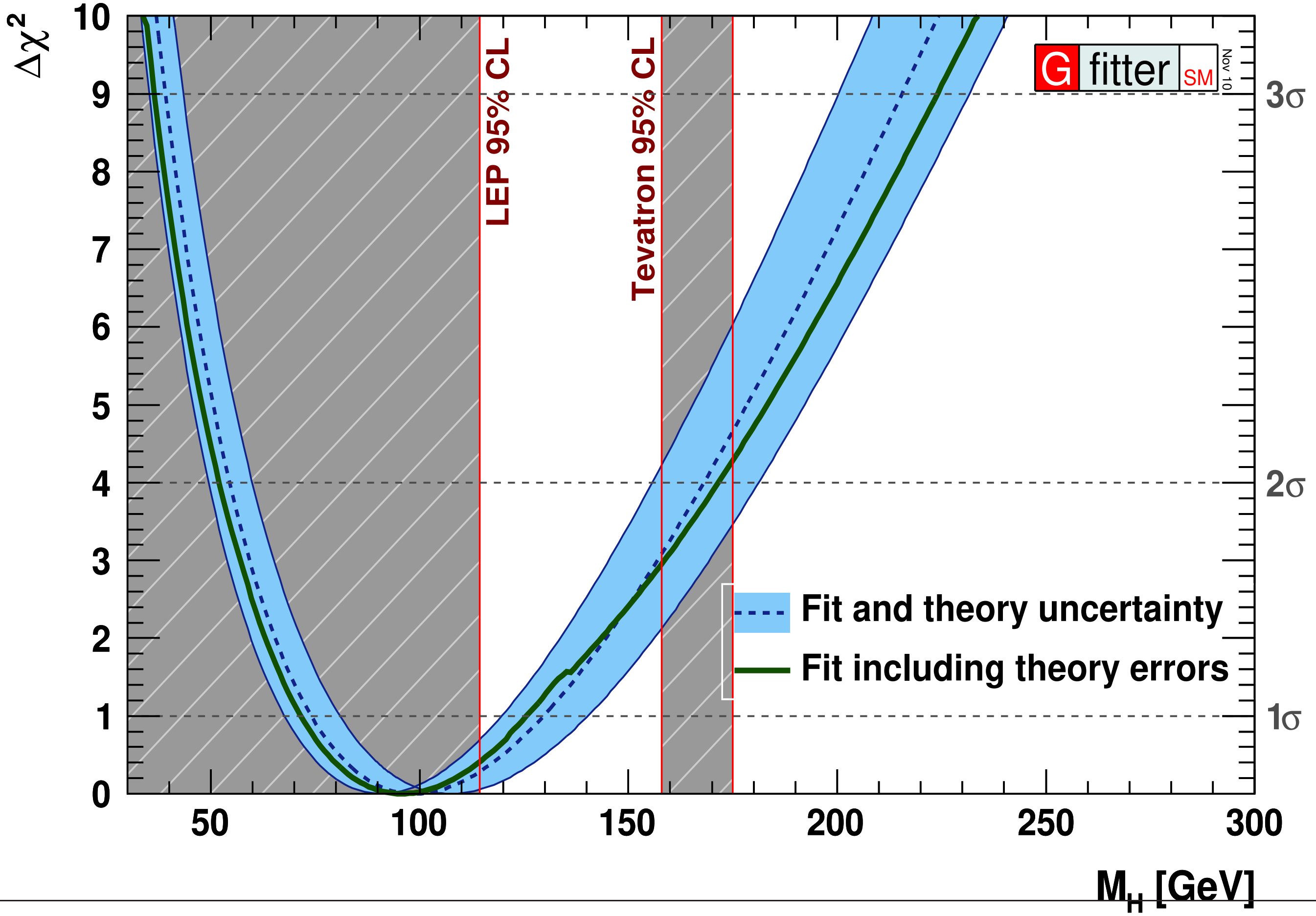
CP violation in meson systems can be explained in terms of the SM parameters and measured CKM mixing in quark sector.

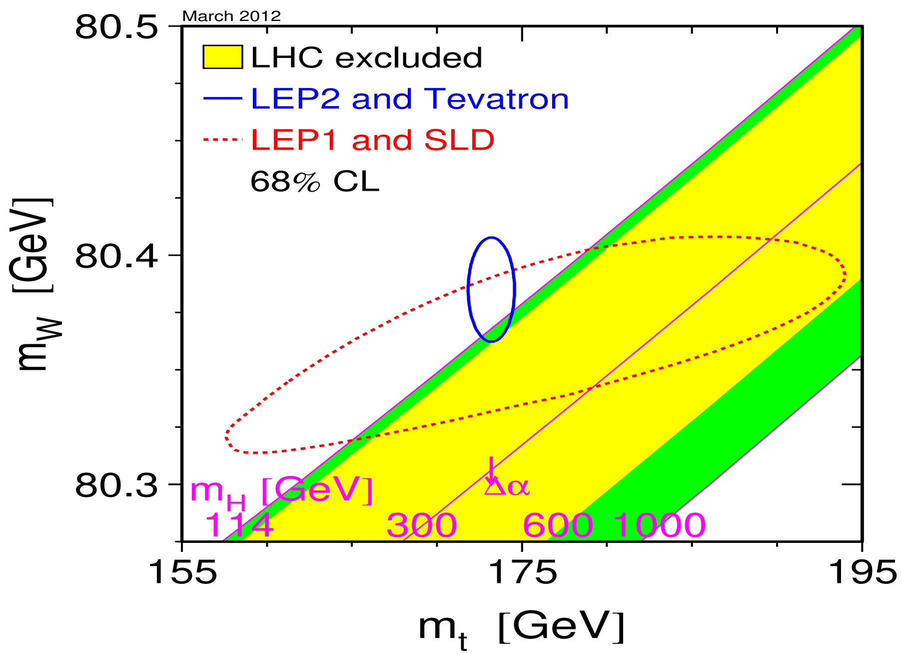
All the neutral current information was explained in terms of a unique value of $\sin\theta_W$ which in turn gave a prediction for M_W, M_Z which was confirmed experimentally.

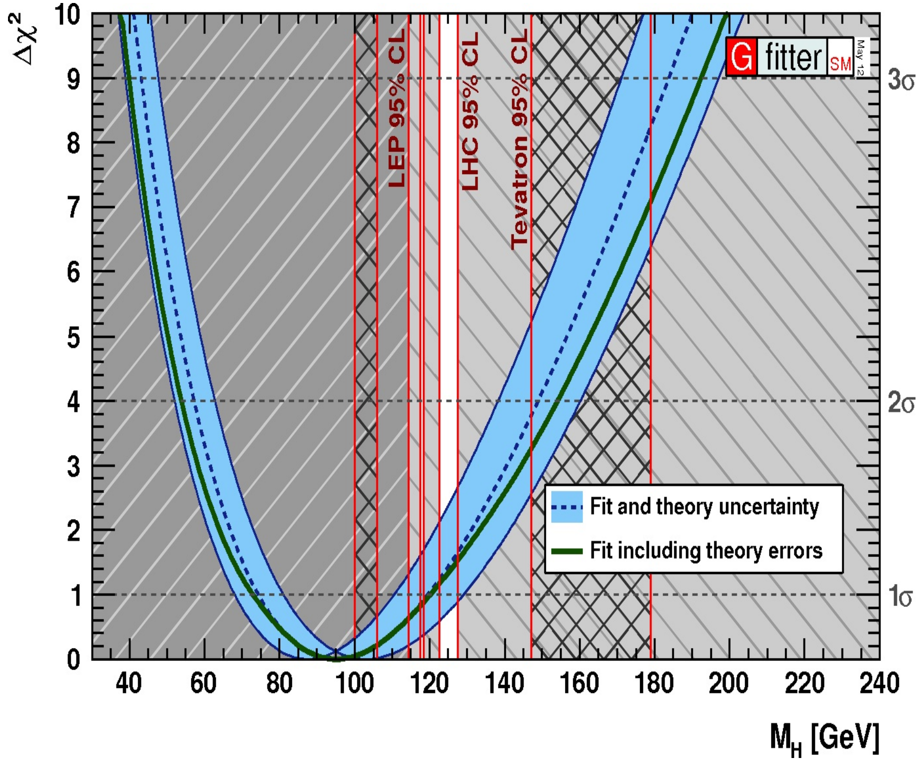
M_t was predicted from precision measurement of M_W, M_Z and agreed with the observed mass.

Prediction for where Higgs mass should be?

Now fast forward to 2011: dawn of Higgs discovery. Higgs mass in the SM should be less than 160 GeV (Indirect information!)







Exptal. Limits: from nonobservation of Higgs in direct searches and indirect limits from LEP/Tevatron precision measurements.

Before the observation of signal at the LHC: **Precision EW measurements like LIGHT Higgs**. For the SM to be correct Higgs HAD to be light!

Did pure theory have anything to say?

The 'direct' limits were from explicit searches.

The 'indirect' limits used theory, but needed the 'precision data'.

But before all existed pure theoretical limits arrived at by demanding theoretical consistency.

Recall λ was a free parameter and SM has no prediction for it!

Dependent on a completely arbitrary parameter, not related at tree level to any other parameter of the Standard Model.

Can some theoretical considerations tell us something about its mass?
Only through the 'indirect' effect.

We already saw how m_h is constrained by precision measurements.

In addition there are pure theoretical bounds.

Three kinds of bounds.

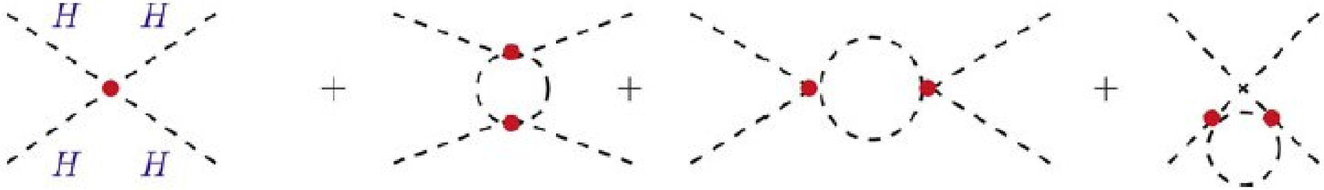
1) Considerations of unitarity of scattering amplitudes.

2) Considerations of quantum corrections to the self coupling λ in the scalar potential.

Unitarity and Triviality considerations give an upper bound whereas considerations of stability give a lower bound.

Triviality and Stability Bounds: demanding that the quartic coupling in the Higgs potential $V_h = \lambda v h^3 + \lambda/4 h^4$ remains perturbative and positive, under loop corrections.

The corrections come from:



When m_h is small and λ not large, the fermion/gauge boson loops are important. Fermions loops come with a negative sign!

Now the RGE for λ is given by

$$\frac{d\lambda(Q^2)}{d\log(Q^2)} \simeq \frac{1}{16\pi^2}[12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2)]$$

λ_t is the Yukawa coupling for the top. At small m_h and hence small $\lambda(v)$, at some value of Q, λ can turn negative. Potential will be unbounded. Vacuum will be unstable.

The condition is

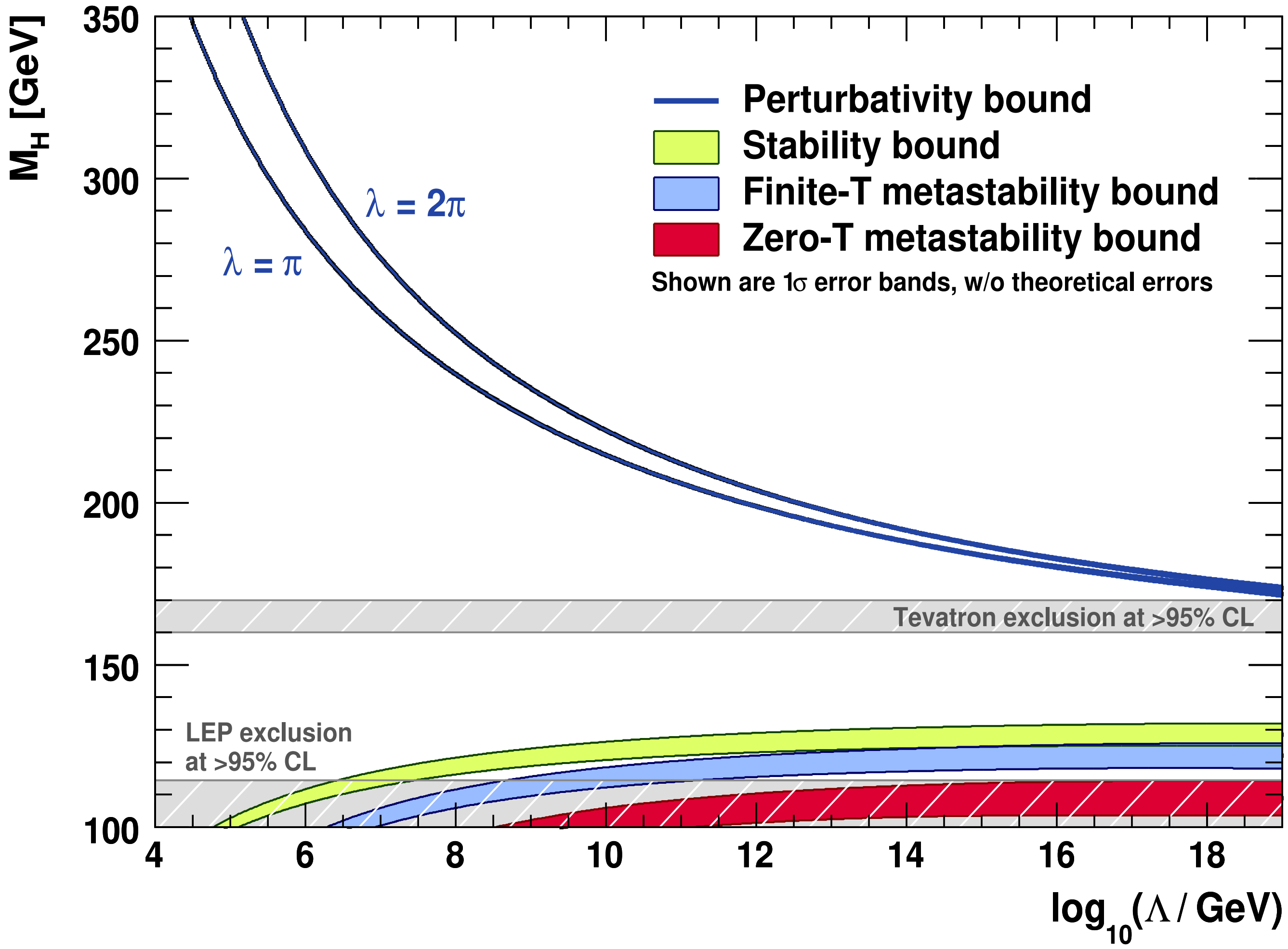
$$m_h^2 > \frac{v^2}{8\pi^2} \log(Q^2/v^2) \left[12 m_t^2/v^4 - \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2) \right].$$

If we demand that the $\lambda(Q)$ is positive upto Λ_C we then get a lower bound.

For example:

$\Lambda_C = 10^3 GeV$, $M_h \gtrsim$ 70 GeV

Earliest calculations of such stability bounds by Linde, Weinberg.

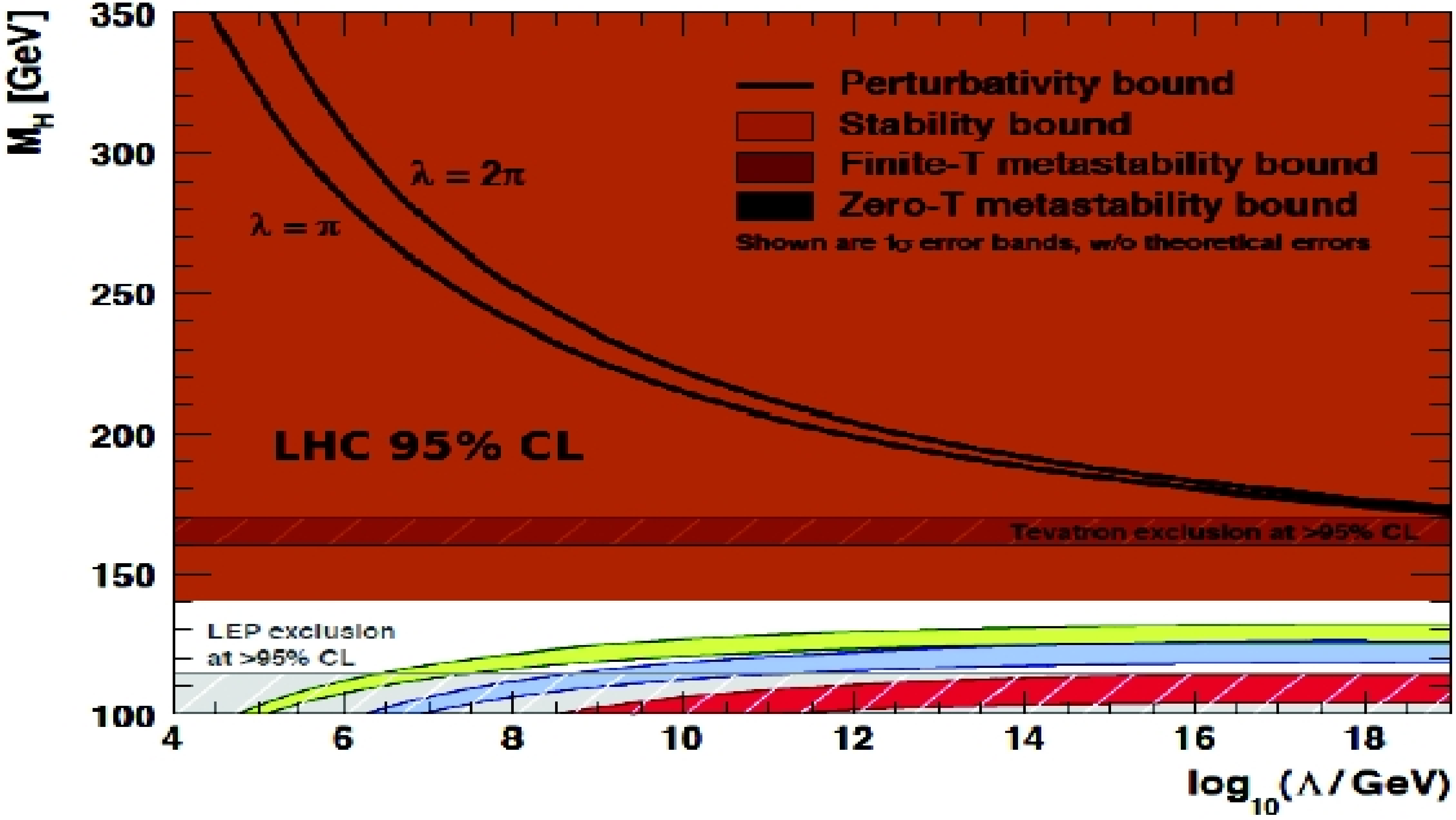


From a paper by Ellis, Giudice et al, PLB 679, 369-375 (2009). Includes higher order effects compared to the formulae here.

In view of the rather small values of m_h indicated by EWPT, need for more accurate calculation of these limits was required.

These limits critically depend also on $m_t^{\bar{M}S}$

State of the art in 2009: (Ellis, Giudice et al:0906.0954)

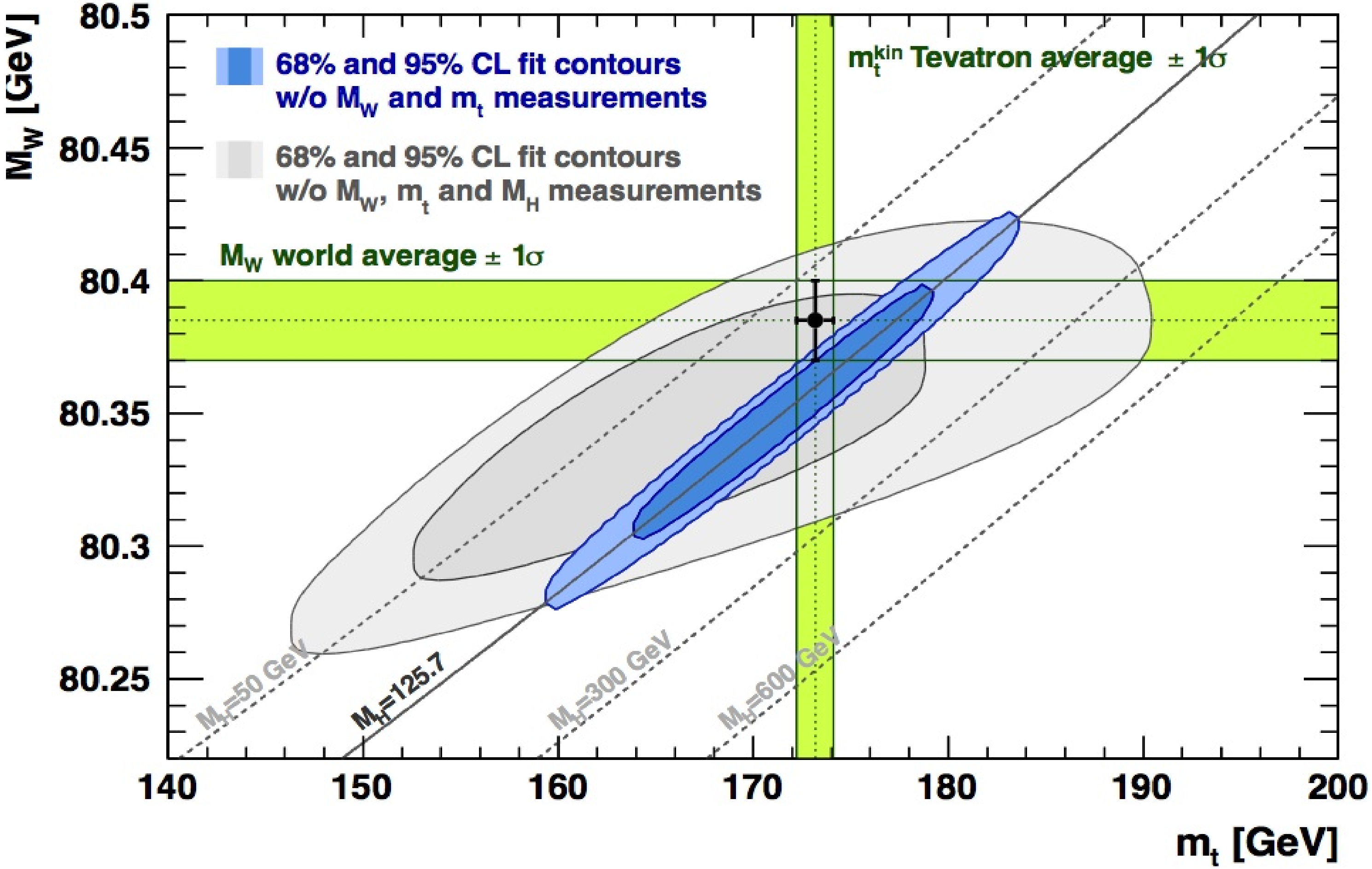


December 2011

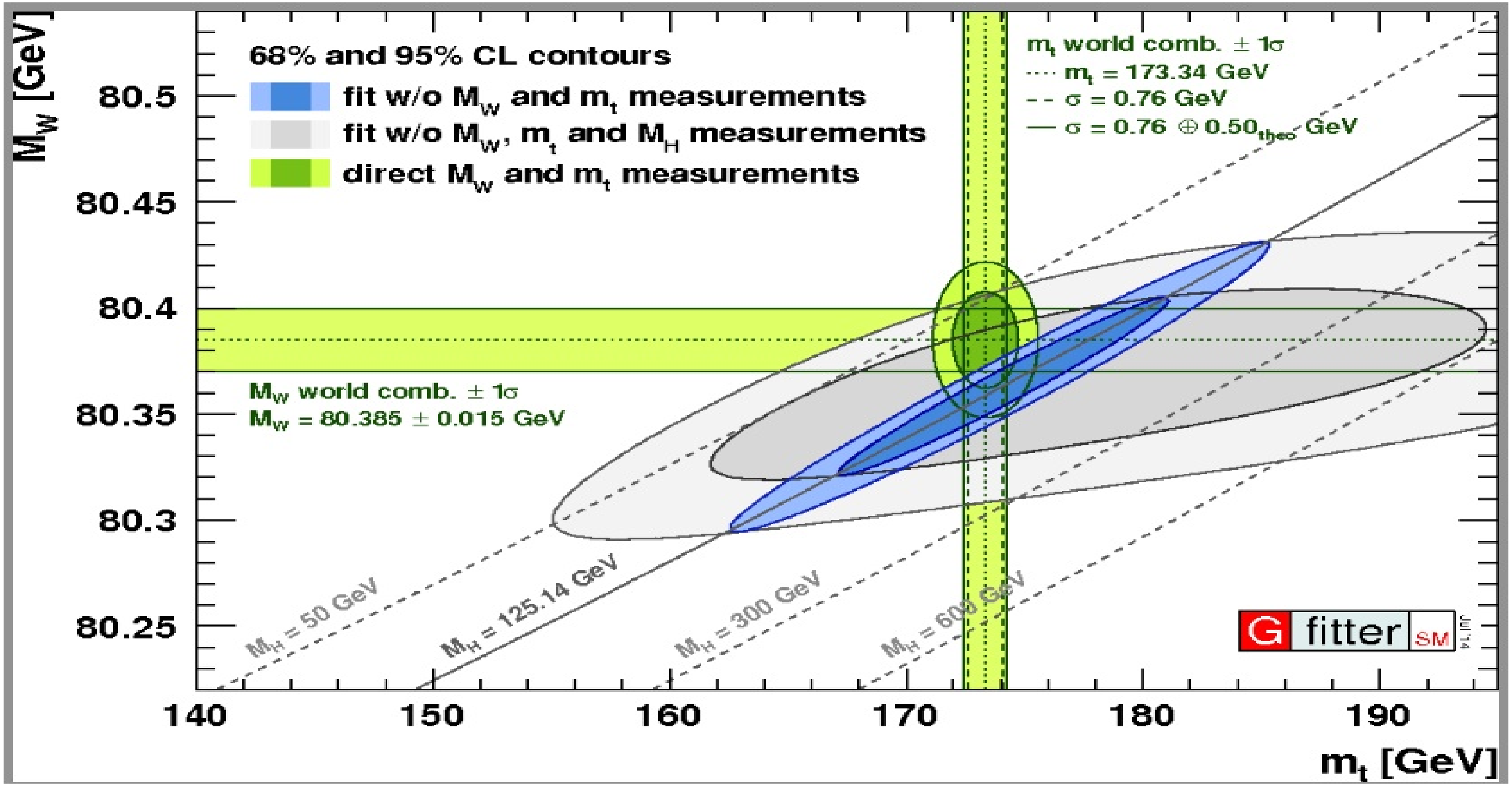
Allowed Higgs mass range was restricted to a very narrow range with all the different constraints. If the Higgs had not been discovered ten years ago where it was, perhaps it would have been even more fun!

But to the relief of all of us who lived through the four decades from 1974 - 2012, LHC did find the Higgs in that 'narrow' slither!

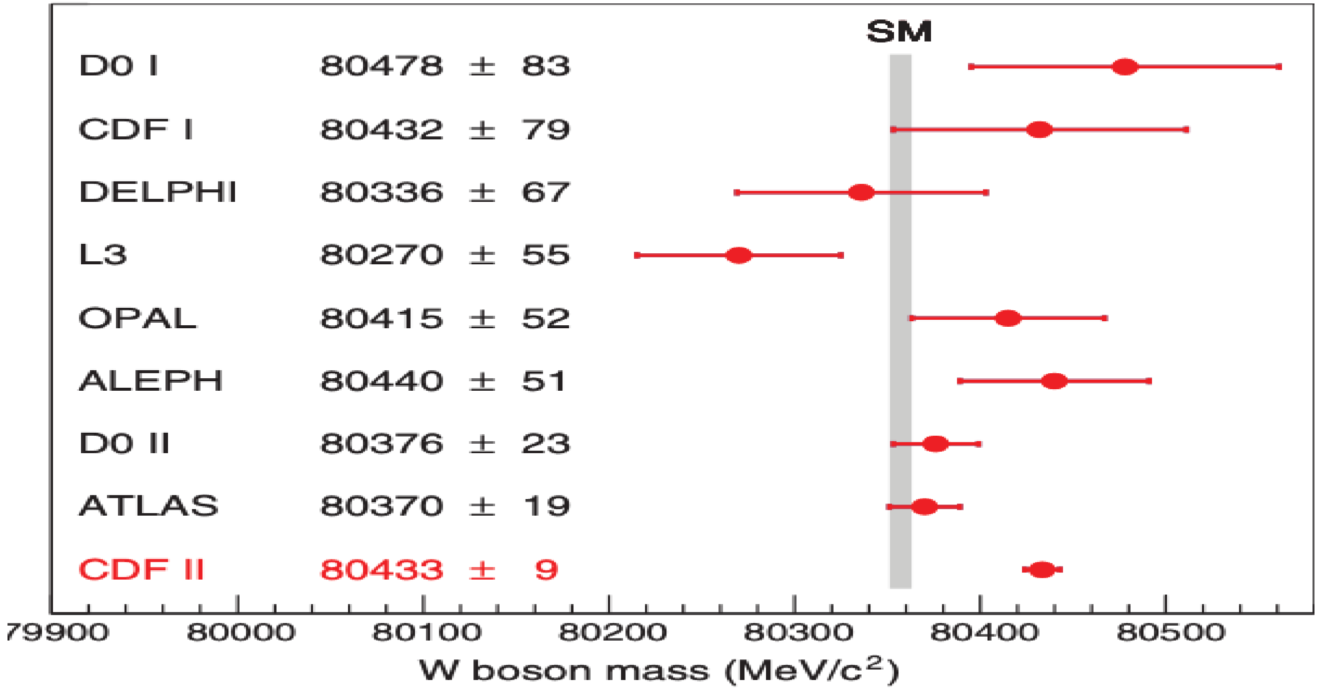
Gave the final confirmation of the 'correctness' of the 'model of leptons!'.Triumph of Gauge principle and SSB!



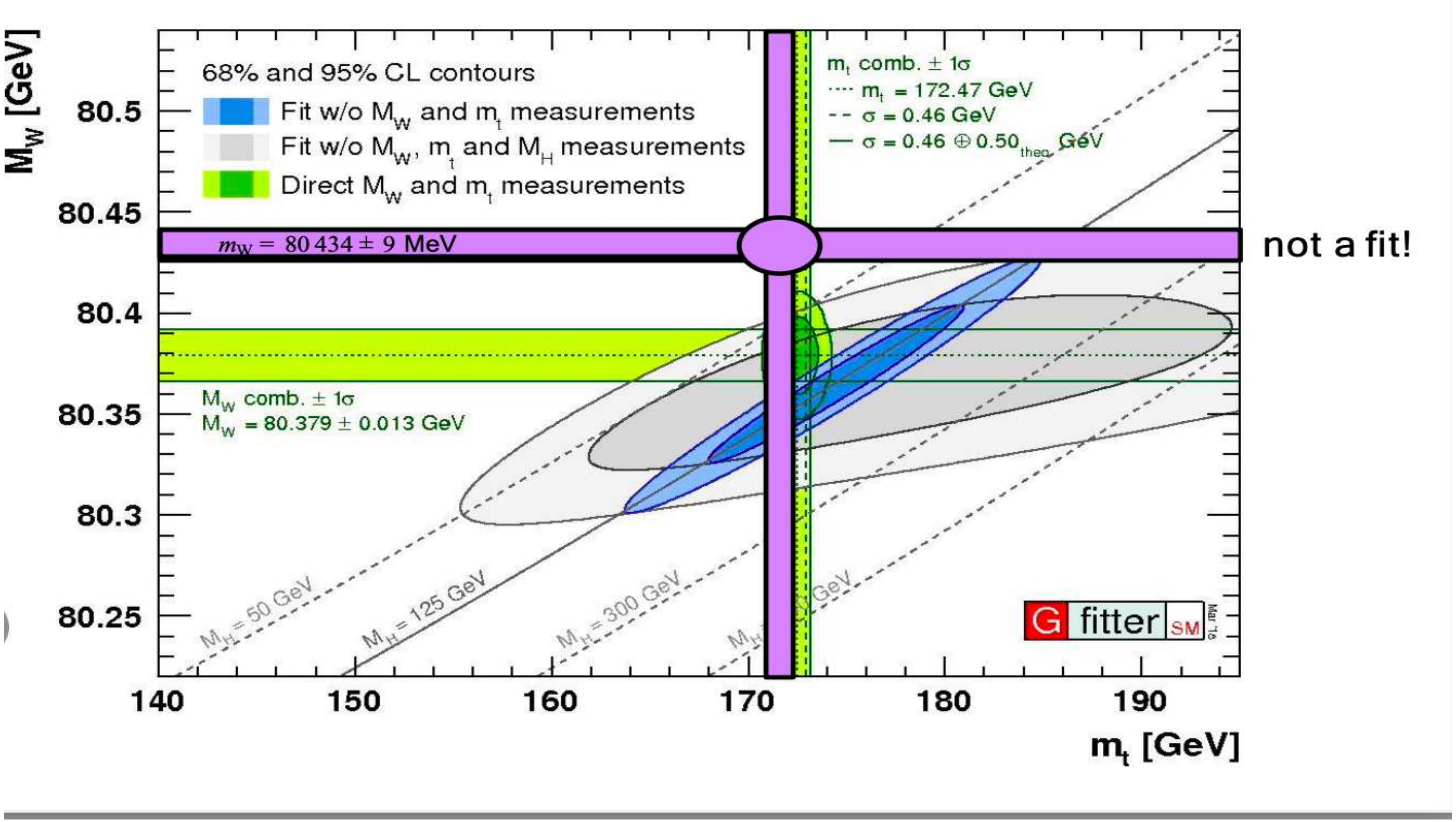
Implication number 1 of the observed m_h value : SM rocks!



SM rocks! At LOOP level. M_W slightly larger than the fit prediction.!



In this case it is the precision of the new experimental result that needs scrutiny!



(From Martijn Mulders).

If the new measurement and its error hold scrutiny and the discrepancy with the SM continues:

1) Missing higher orders? (unlikely)

2) Virtual effects of New particles and interactions? (SUSY?)

3) Tree level effects of additional Higgses? (Triplet?)

For 2 and 3: one has to be consistent with non observation of any new physics so far!

But first we wait to see if the error in CDF result is as small as it is reported!

Just the mass of m_h can give indication of the scale of new physics beyond the SM.

For example: It already tells that SUSY scale needs to be large if SUSY is relevant to particle physics!

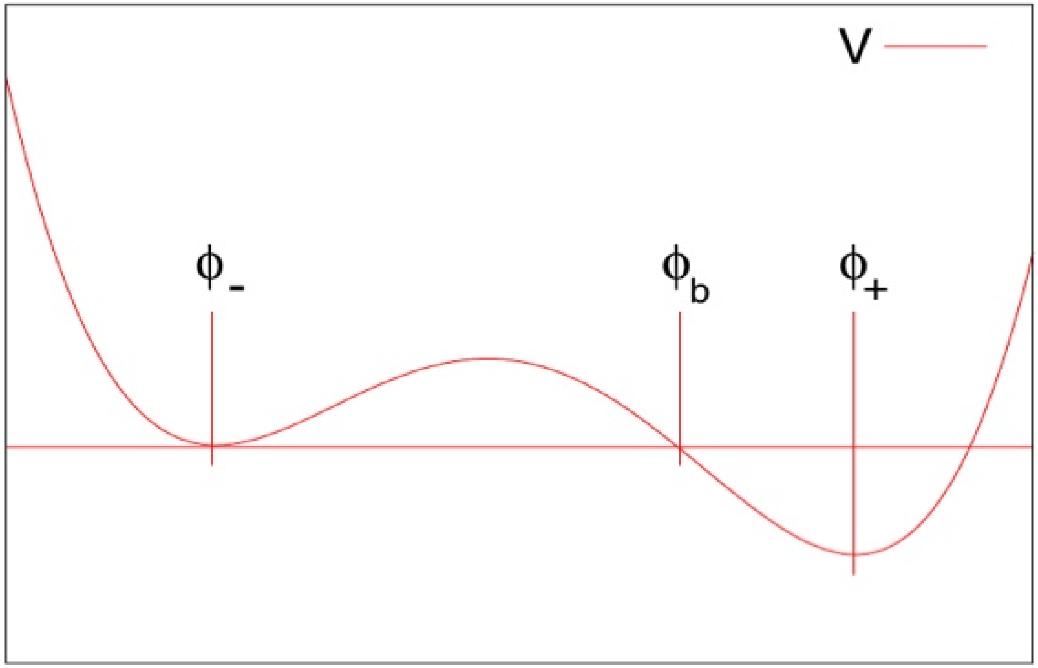
But are there ANY theoretical consistent arguments that tell us if SM is all that there is upto Planck scale or is there something something more?

Can we see any footprints (say) in the masses and properties of the two heaviest particles h and t?

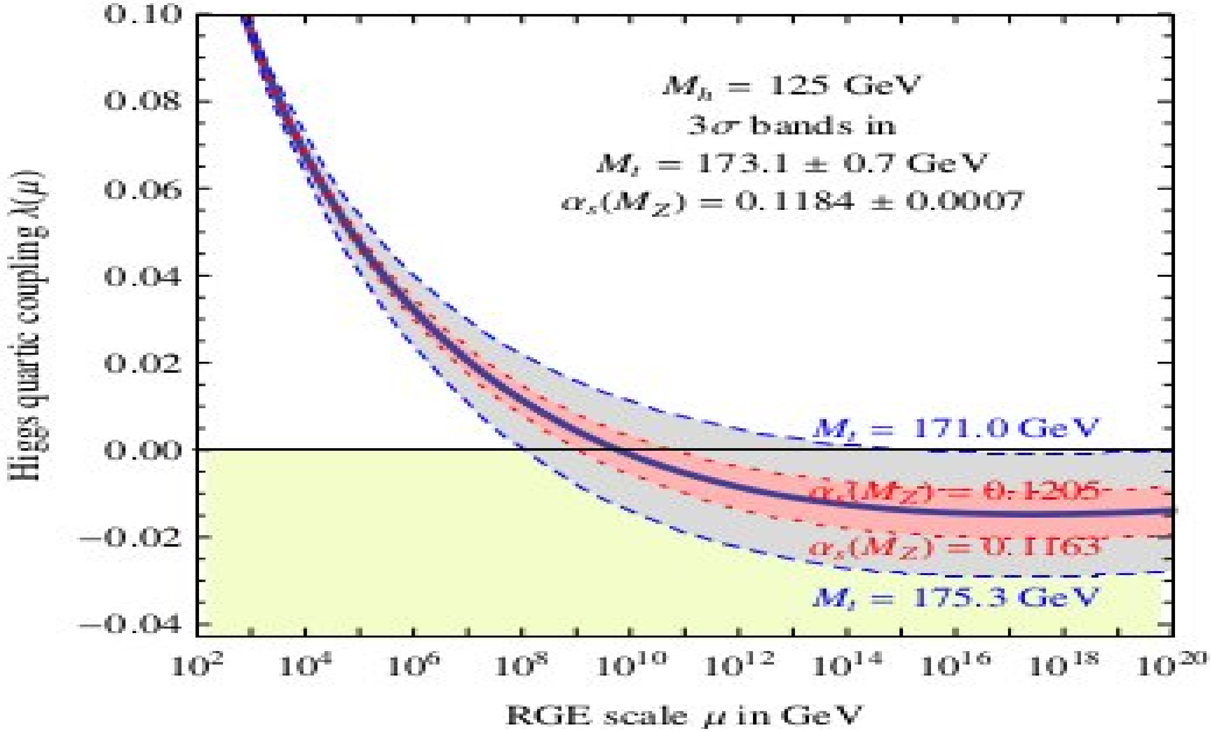
There masses can carry some message about the BSM!

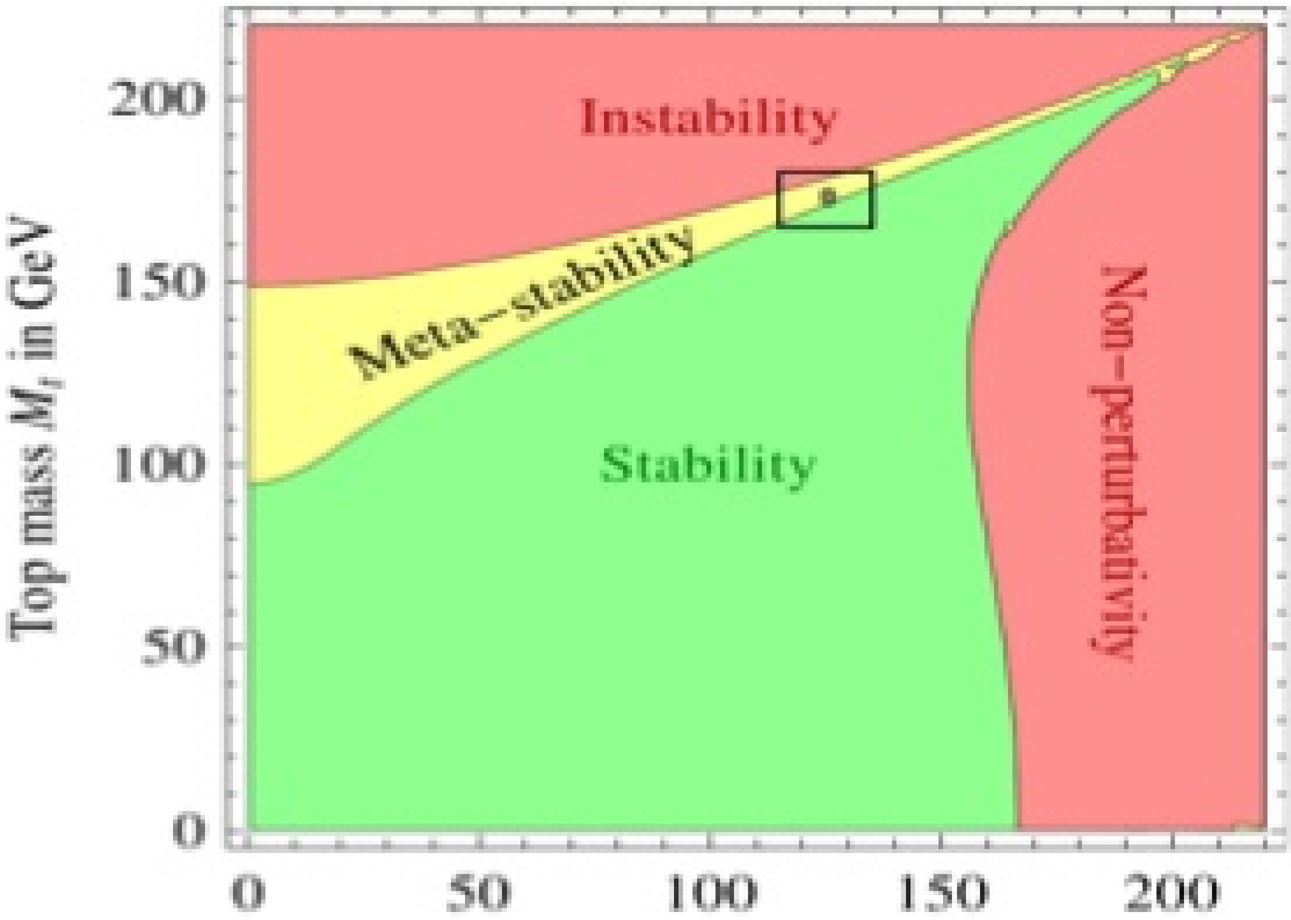
Planck scale dynamics might stabilise the vacuum for $|\Phi| >> v$ and we might be living in a metastable vacuum which has a life time bigger than that of the Universe.

How to calculate transition rates: Coleman showed us in 1977!



Increase the accuracy of the calculations of 'instability'. De Grassie et al (1205.6497) Complete NNLO analysis. Major progress. Theoretical error on the obtained bounds due to missing higher order corrections reduced to 1 GeV

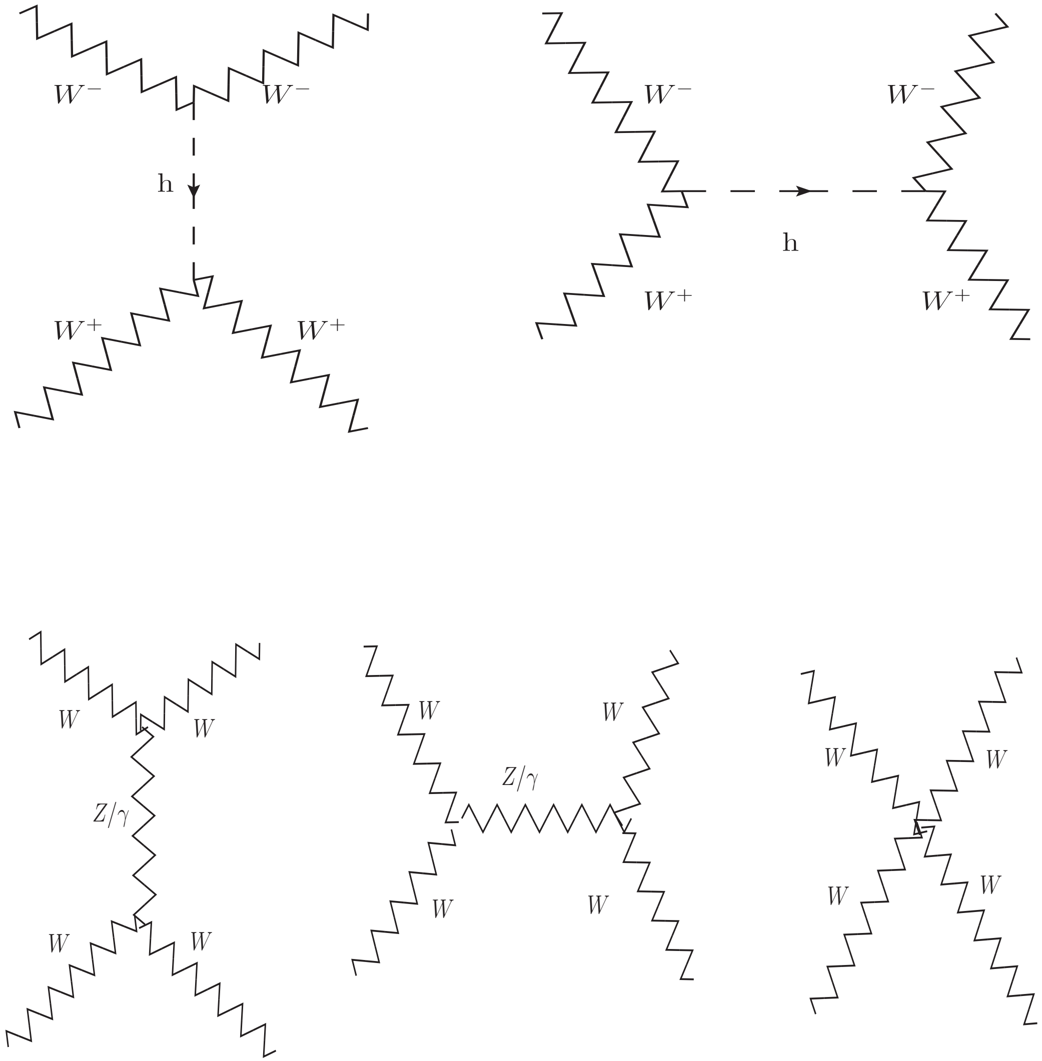



Instability
Meta–stability
Stability
Non–perturbativity
Top mass M_t in GeV
200
150
100
50
0
0
50
100
150
200

Upper limits on m_h not very interesting anymore as the observed Higgs is 'light' and hence 'weakly coupled'.

When we did not have any information these theoretical upper bounds served a very useful purpose.

In fact was it helped justifying the LHC project and set the design goals for LHC such that it can cover the entire range allowed theoretically.



Each of these amplitudes has bad high energy behaviour, $\propto s^{\alpha}$, $\alpha = 1/2$.

These cancel against each other ONLY for couplings of the Z and h with the W pair as expected in the SM.

If there exist heavy fermions then one needs the $hf\bar{f}$ couplings to be exactly as in the SM!

Looks like there are deep relationships between unitarity and renormalisabilty.

C. Quigg, H.B. Thacker and B.W. Lee, Phys. Rev. D noticed the following:

Cancellation of leading divergencies is independent of the mass of the Higgs, but the $WW \to WW$ amplitude has a m_h dependent piece

$$\mathcal{A}(W_L^+ W_L^- \to W_L^+ W_L^-) = -\sqrt{2} G_\mu M_h^2 \left(\frac{s}{s - M_h^2} + \frac{t}{t - M_h^2} \right).$$

This will violate unitarity for $m_h > \left(\frac{8\pi\sqrt{2}}{3G_\mu}\right)^{1/2} \sim$ 1000 GeV. A more correct calculation keeping the logarithms gives 780 GeV.

The small value of the mass of the observed state means that the SM satisfies tree level unitarity without any trouble!

At large m_h and large λ considerations of triviality give an upper bound. That used to be of great concern !

Remember: $m_h^2 = \lambda v^2$. For large λ the loop corrections dominated by the h–loops.

At one loop running of λ given by:

$$\frac{d\lambda(Q^2)}{d \log Q^2} = \frac{3}{4\pi}\lambda^2(Q^2)$$

Solving this, one gets

$$\lambda(Q^2) = \frac{\lambda(v^2)}{[1 - \frac{3}{4\pi^2}\lambda(v^2)\log(\frac{Q^2}{v^2})]}$$

For large $Q^2 \gg v^2$ then $\lambda(Q^2)$ develops a pole (the Landau pole).

If we demand that λ remain always in perturbative regime, we can ONLY have $\lambda = 0$. Theory will be trivial.

One can take an alternate view:

Demand that the scale at which λ blows up is above a given scale Λ.

For a given m_h the scale at which the pole lies

$$\Lambda_C = v \ \exp\left(\frac{2\pi^2}{3\lambda}\right) = v \exp\left(\frac{4\pi^2 v^2}{3M_h^2}\right)$$

Using $\Lambda_C = \Lambda = 10^{16}$ GeV, we will find $m_h \lesssim 200$ GeV. **Upper Bound: called triviality bound**

Just the mass of m_h can give indication of the scale of new physics beyond the SM

1) g-2 of the muon

2) Flavour non universality

3)Precision measurement of the mass of the W.

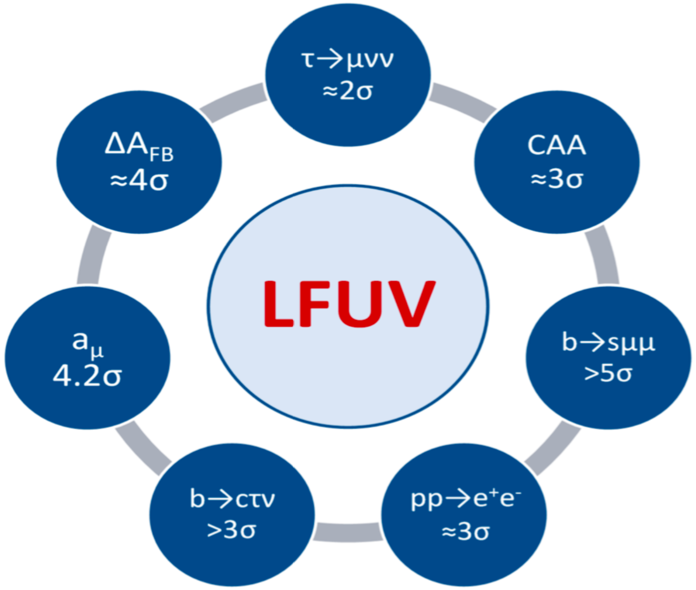
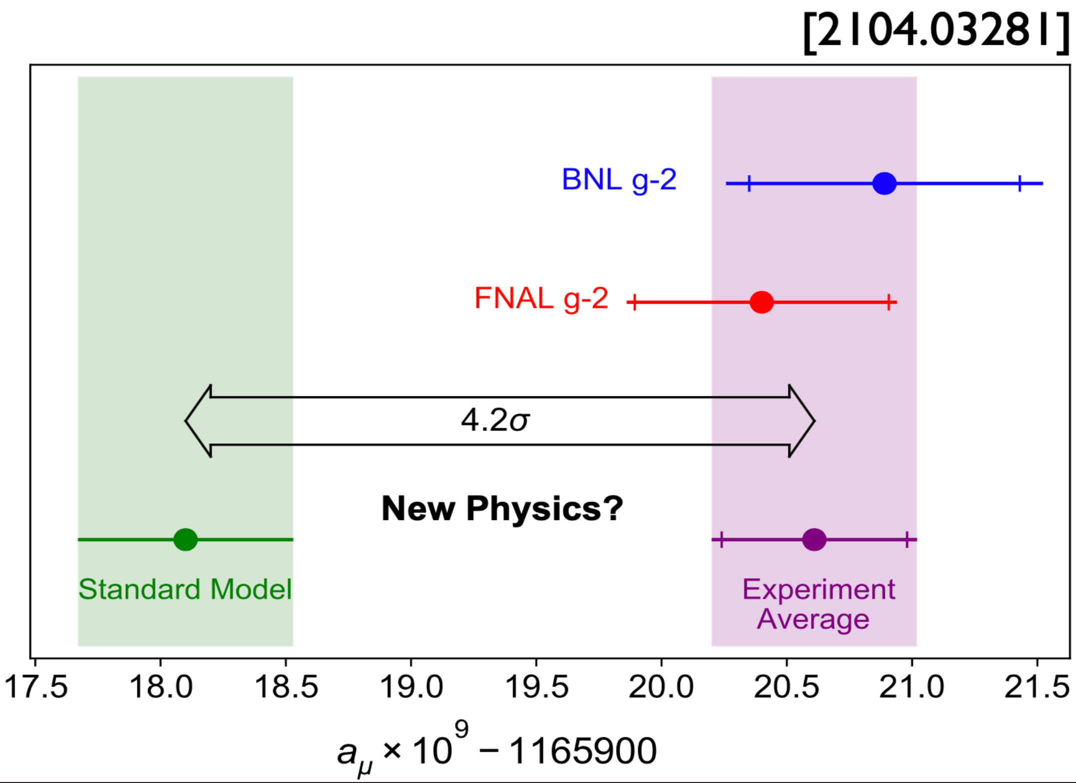


Figure 1: Summary of the experimental hints for LFUV beyond the SM.

Lepton Flavour Universality Violation: is that the thin edge of the wedge? Ref: Unveiling Hidden Physics at the LHC [arXiv:2109.06065 [hep-ph]], Eur. Phys. J. C **82**, no.8, 665 (2022) Sure to be discussed in flavour physics course.

Difference between the theoretical expectation in the SM and measured value at one ppb. (Phys. Rev. Lett. 126, 141801). Evidence for new physics seems compelling. Critical evaluation of theory 'prediction' necessary.



In any case the days of Standard Model are coming to an end in some sense!

Hopefully the case will be 'The King is Dead', 'Long live the King'!

Already the mass of the observed state can be used to answer the question about the scale unto which the SM is valid.

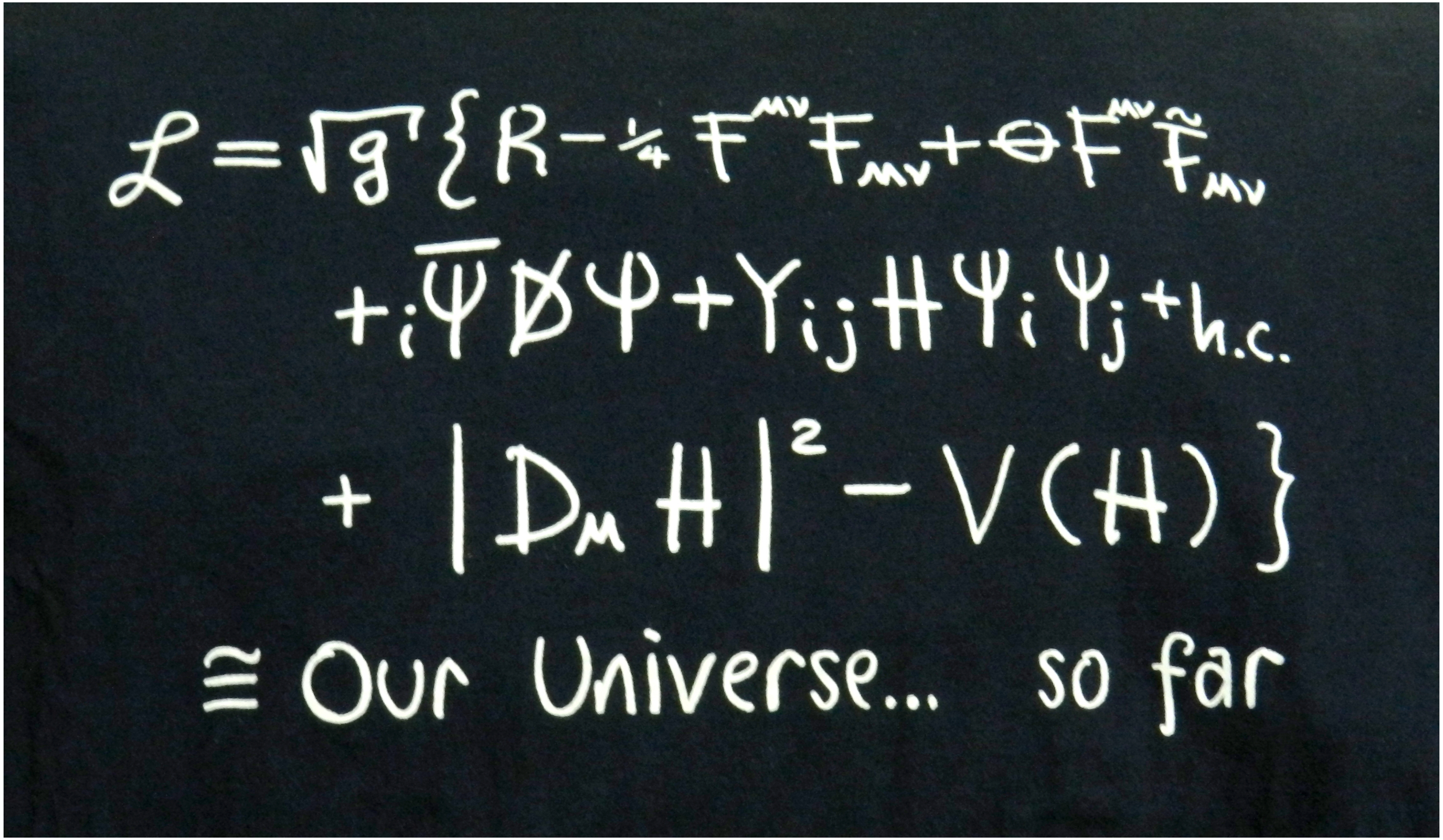
Just like the **gauge principle** and the **unitarity** were the guiding principle so far now the **'light' scalar** might be the guiding principle for future developments!

We should get a peek at the BSM land through the 'window' of measurement of the properties of the Higgs!

Exciting days ahead for sure!

If 14 TeV LHC should also fail to find 'direct' evidence for the BSM physics we would really have to understand what is so special about the Standard Model and hopefully that answer won't be Anthropic Principle !


BSM
HIGGS


$\mathcal{L} = \sqrt{g}\,\{ R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \theta F^{\mu\nu} \tilde{F}_{\mu\nu}$
$+ i \bar{\Psi} \not{D} \Psi + Y_{ij} H \Psi_i \Psi_j + h.c.$
$+ |D_\mu H|^2 - V(H) \}$
$\cong$ Our Universe... so far

1)Gauge Theory of Elementary Particle Physics, T.P. Cheng and Ling-Fong Li

2)M. Peskin and S. Schröder, An Intorduction to Quantum Field Theory

3)The Standard Model, C. Burgess and G. Moore

4)Gauge Theories of Weak Inteactions: W. Greiner and B. Mueller

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