How do we know what lies within?

Rohini M. Godbole
Centre for High Energy Physics, IISc, Bangalore, India

Teachers Training Program, IISER-TVM.

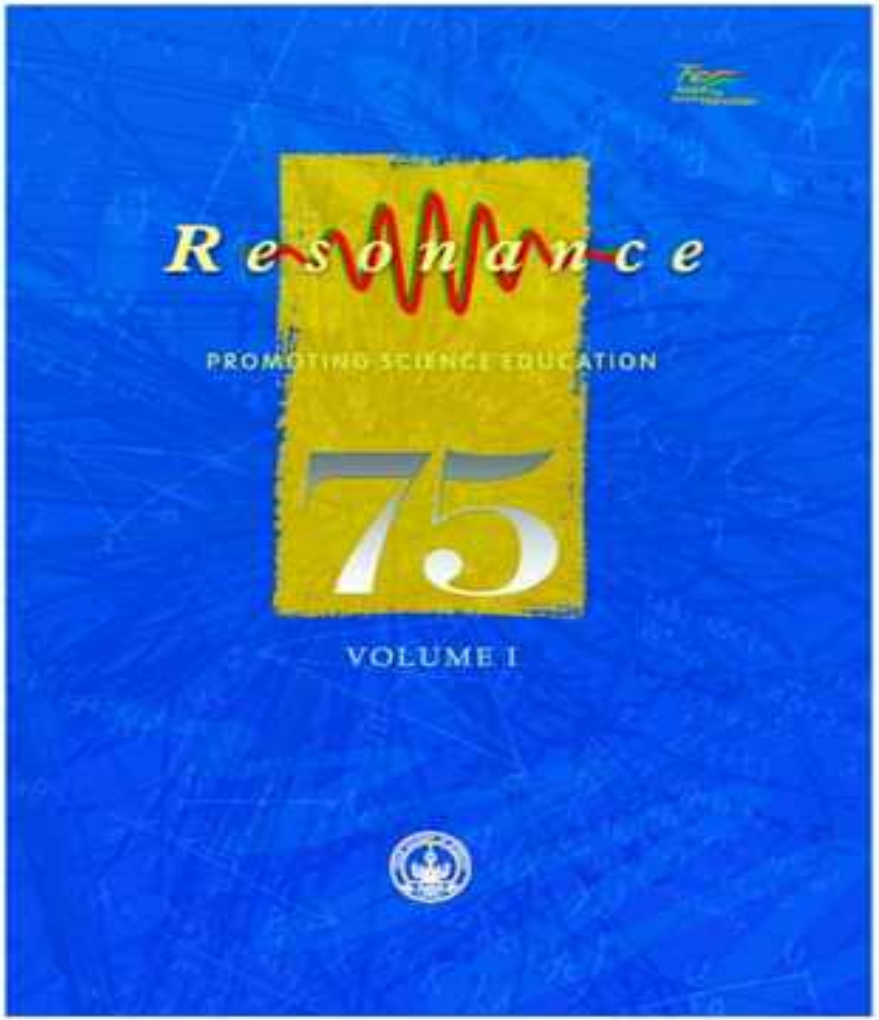
Ref:

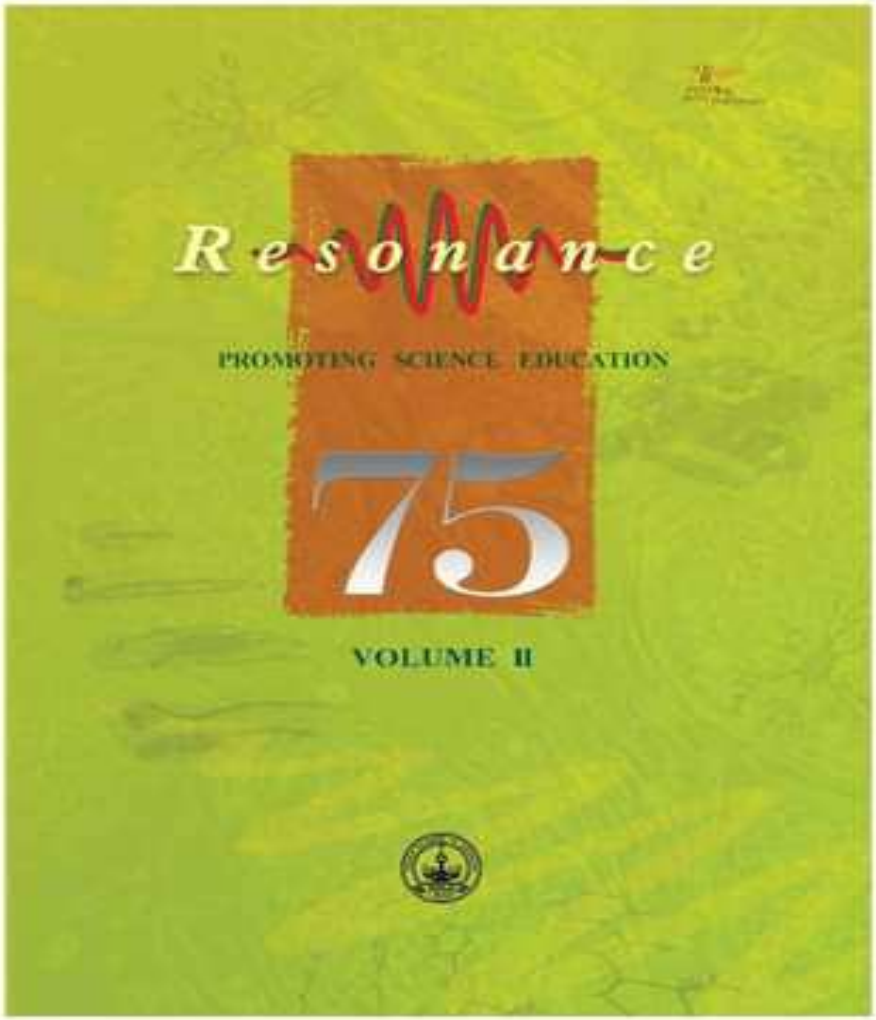
1) "The experiment that shaped the physics of the century," Resonance, Vol.16, No.11, pp.1019–1028, 2011 .

1') An expanded version of the "The experiment that shaped the physics of the century", Resonance-75, Volume 1, page 361, 2022.

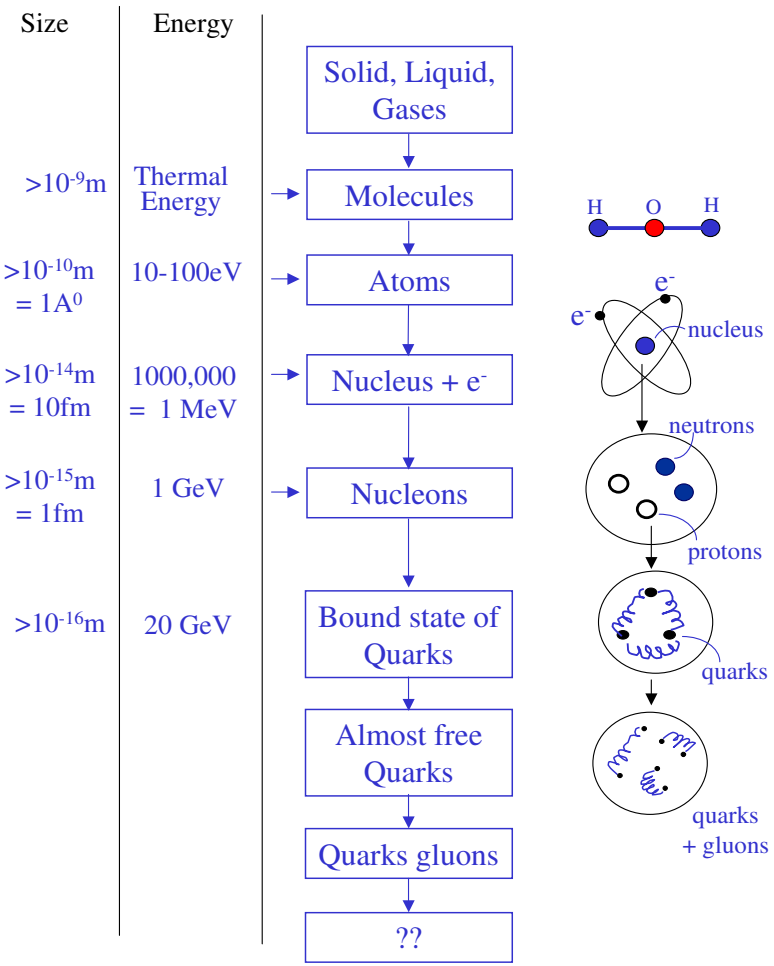
2) e-Print: 1007.0946 , "From Rutherford to LHC and story onwards",

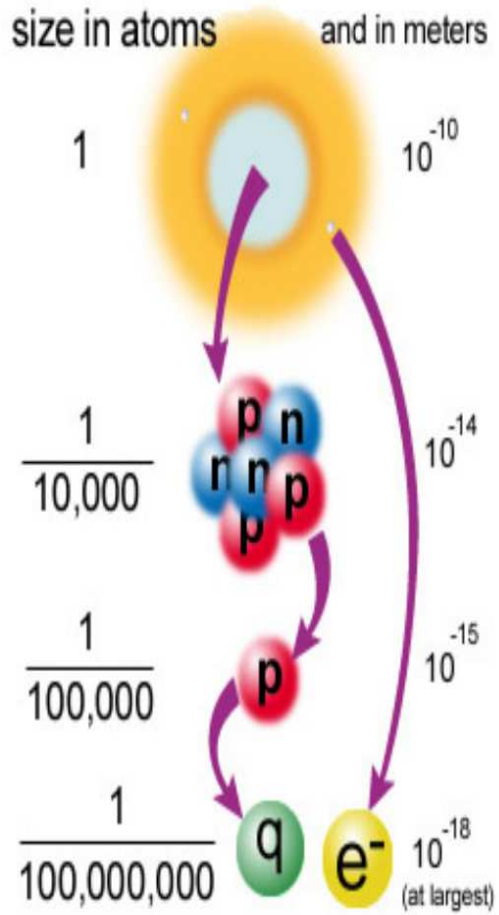
3) 'Saga of the Periodic Table of the Standard Model', Physics News Vol. 51, 34, (2021).





Collection of 75 articles from Resonance.


Size
Energy
Solid, Liquid, Gases
>10-9m
Thermal Energy
Molecules
>10-10m = 1A0
10-100eV
Atoms
>10-14m = 10fm
1000,000 = 1 MeV
Nucleus + e-
>10-15m = 1fm
1 GeV
Nucleons
>10-16m
20 GeV
Bound state of Quarks
Almost free Quarks
Quarks gluons
??
H O H
e-
e-
nucleus
neutrons
protons
quarks
quarks + gluons

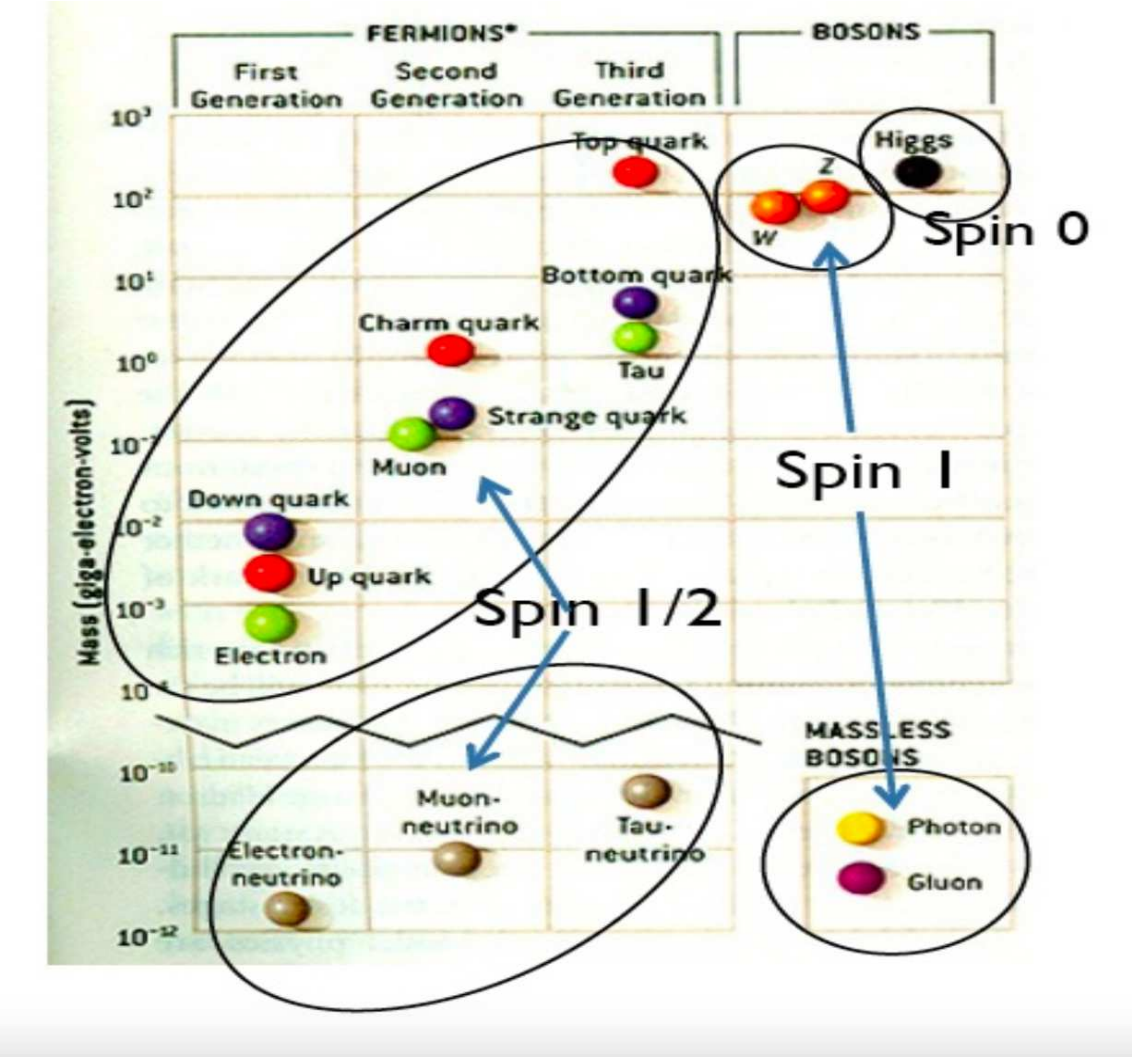

size in atoms
and in meters
1
10-10
1/10,000
10-14
1/100,000
10-15
1/100,000,000
10-18
(at largest)
p n
n n p
p
p
q
e-

How important is the idea of 'atoms'?

In the words of R.P. Feynman:

" *If all the scientific knowledge in the world were to be destroyed and I can choose only one piece of understanding to be passed on to future, I would choose to pass on the message that matter is composed of atoms, ceaselessly moving and bouncing against each other.*"

SM matter particles and force carriers



Particle content of the STANDARD MODEL (SM) OF PARTICLE PHYSICS!

The 'Periodic Table' of Fundamental particles and their interactions has arrived!

Addition of gravitational interaction and spin-2 graviton will complete the picture!. We only touch upon it here.

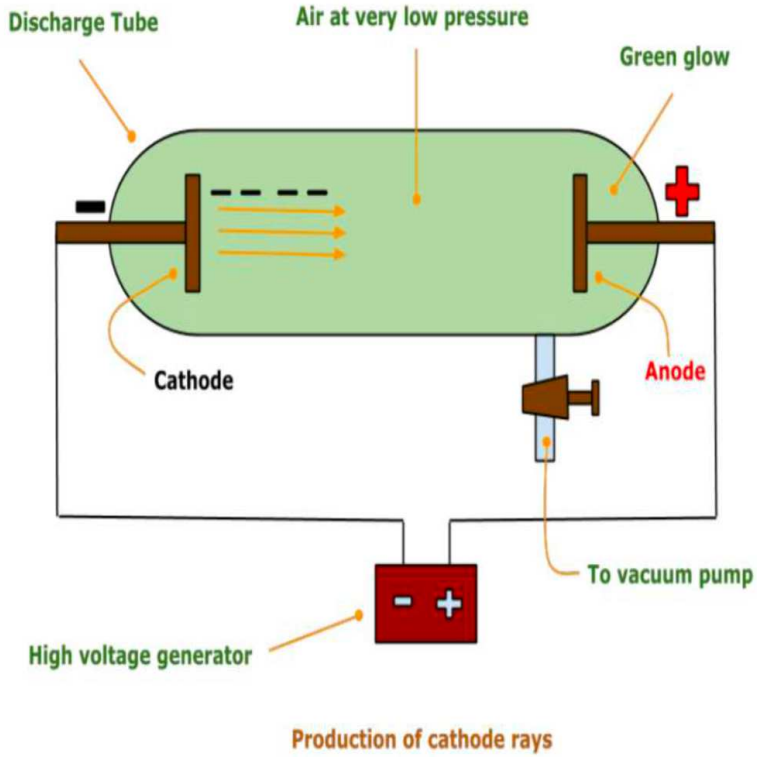
Intermediate stops at the elementarity station:

1)Nucleus: 'discovered' by Rutherford in α particle scattering (1911). This truly began the inward bound journey into the heart of matter!

2)Pions : predicted by Yukawa (1935), found in cosmic ray experiments (1947).

3)Existence of protons was inferred from observations and that of neutrons from properties of nuclei and then neutron was discovered in 1932.

These were considered elementary at these intermediate points in the journey.



Second half of 19th Century:

Faraday: electricity too comes in multiples of a basic unit.

$\Downarrow$

Experiments with Cathode Rays by Thompson and Discovery of the electron: A particle with e/m ratio different from the hydrogen ion. **The first elementary particle.**

1897: In this discovery of the electron by Thompson, the world of elementary particles of today was born.

Three basic processes in the transition from electron being a "postulated entity" to a "physical reality" :1894.. $>$ 1899**.**

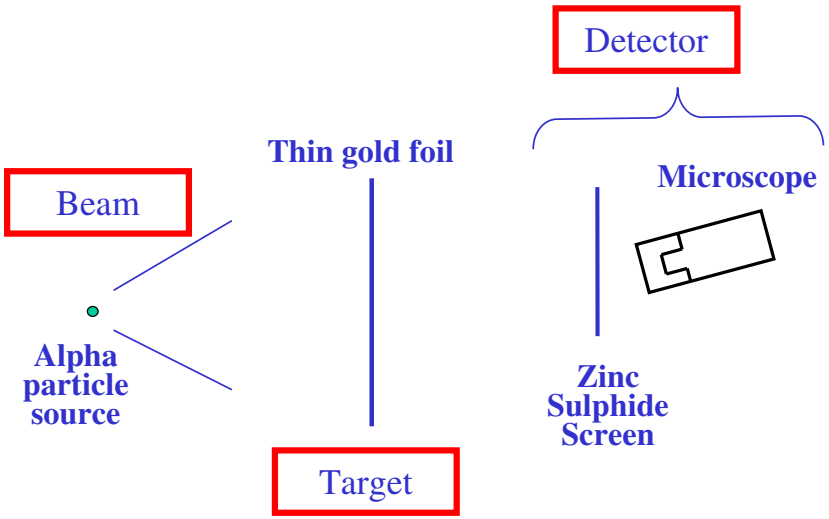
1) Observation by Farady that the electricity comes in units from patterns in ionisation,

2) The experiments made by Thompson that Cathode rays behave under the action of electric and magnetic fields as though they consisted of particles with a ratio of charge to mass (the famous e/m) quite different from the Hydrogen ion,

3) Lorenz calculated splitting of the atomic spectral lines in a magnetic field. The prediction agreed with Zeeman's measurement if value of e/m was equal to that found by Thomson! The 'corpuscle' seen by Thomson in his Cathode Ray Tube was the same that exists in an 'atom'.

Thompson: Plum pudding model of Atom with electrons sticking out like plums.

The Rutherford scattering experiment: shaped the physics of the Century!



of α particles scattered from the gold foil at different angles were counted.

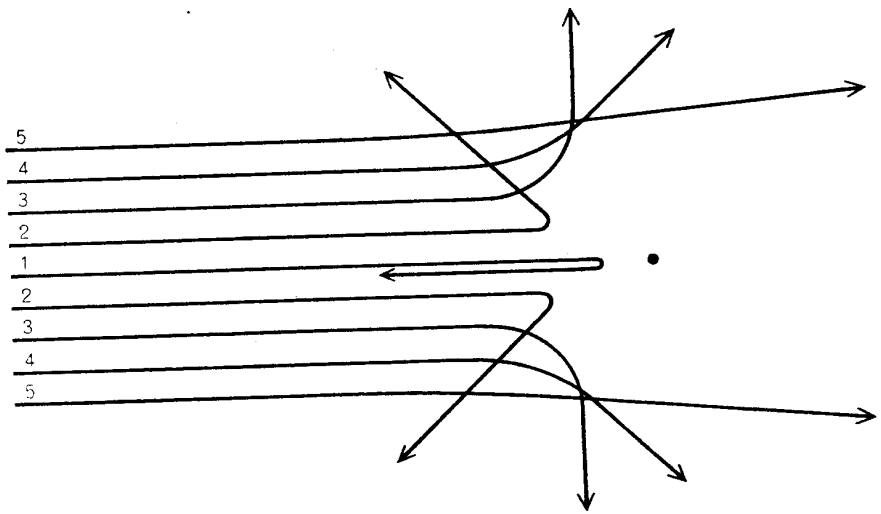
Most α particles went undeflected.

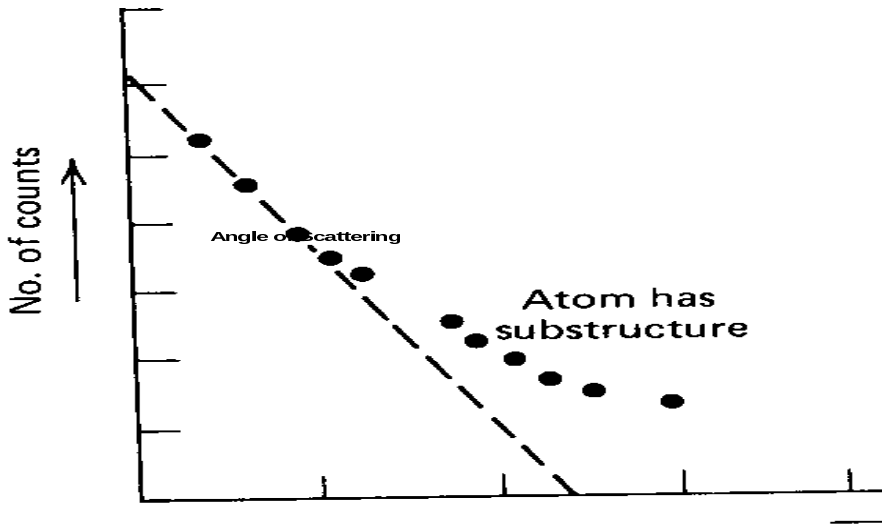
BUT SOME REBOUNDED

Completely opposite to that expected if 'plum pudding model' was true.

Rutherford concluded from this: atom has a point like nucleus.

Rutherford truly split the atom into nucleus and electrons! Why did this mean that positive charge of the atom is a 'point': the nucleus?





Distance of closest approach will be where the α particle comes to a stop and then rebounds. For given energies of α particle the distance was about 60 Fermi. So the charge was concentrated in region smaller than that!

Rebound velocity also gave an estimate of the mass at the centre!

Weinberg, Modern Physics.

Let us work it out more or less from the information available to Rutherford

Why did backward scattering immediately that something much heavier than the electron was repelling the α particle.

Consider particles A, B with masses m_A, m_B colliding head on velocities v_A, v_B and emerging after collision travelling with velocities v'_A, v'_B along the same line.

v_i are velocities. So they can be negative or positive. If both the velocities have the same signs the two particles are going in the same direction, if they are going in opposite direction the velocities have opposite signs.

Momentum and energy conservation requires

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B, \quad (1)$$

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2 \quad (2)$$

Solve the two equations. Substitute v'_B in terms of the other three using equation 1.

Get a quadratic for v'_B. Trivial solution will be $v_A = v'_A, v_B = v'_B$. Nontrivial solution will be

$$v'_A = \frac{(m_A - m_B)v_A + 2m_B v_B}{m_A + m_B}$$

and similarly for v'_B by $A \leftrightarrow B$.

For Rutherford's experiment, $v_B = 0$ and $v'_A = \frac{(m_A - m_B)v_A}{m_A + m_B}$.

Backward scattering means $m_A < m_B$. That means α particle is repelled by something heavier than itself.

Rutherford could estimate that the Nuclear size is much smaller than atomic size by the following argument.

As the α particle approaches the positive charge Z it feels the repulsive Coulomb repulsion.

It will slowly lose its K.E. and will come to a stop depending on the impact parameter and then rebound.

$$\frac{1}{2}m_\alpha v_\alpha^2 = \frac{e_\alpha\, Z_{target}\, e}{r} \Rightarrow r = 2Z_{target}\, e\, \frac{e_\alpha}{m_\alpha} v_\alpha^2$$

Rutherford quotes $v_\alpha = 2.09 \times 10^9$ cm/sec, $e_\alpha/m_\alpha = e/2m_p$.

We get $r = 3Z_{target}\, 10^{-14}$ cm. Even with $Z = 100, r = 10^{-12}$cm.

Much greater than 10^{-8} cm which was estimated to be atomic size from the density of metals and known atomic masses!

Study the distribution of the counts as a function of scattering angle and compare it with that expected from a point target. If they differ the target has a structure.

Simple calculation showed that for a point, screened charge one would expect:

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{sin^4(\theta/2)}$$

Modification from this then can be used to extract information about the 'spatial extent (the 'size') of the target.

How do we measure sizes and distances?

I want to discuss how we measured, e.g., the size of nuclei.

Most probably you have heard about Hofstadter Expt. and taught in Nuclear Physics class that the radius R of a Nucleus is $R_0 A^{1/3}$ where $R_0 = 1.4$ fm.

What physical process was used?

Why did this measurement reveal existence of a finite size of nucleus?

How come Rutherford concluded that the charge in an atom is in a region smaller than 10^{-12} cm.?

This would also give an idea why higher & higher energies were required to reveal objects with smaller and smaller sizes.

Reminder

When we are using (say) microscope to increase resolving power we decrease the wavelength λ.

If $\lambda \gg e$ (the size of object)

then the object will be seen only as a "point"

Remember also De Broglie relation

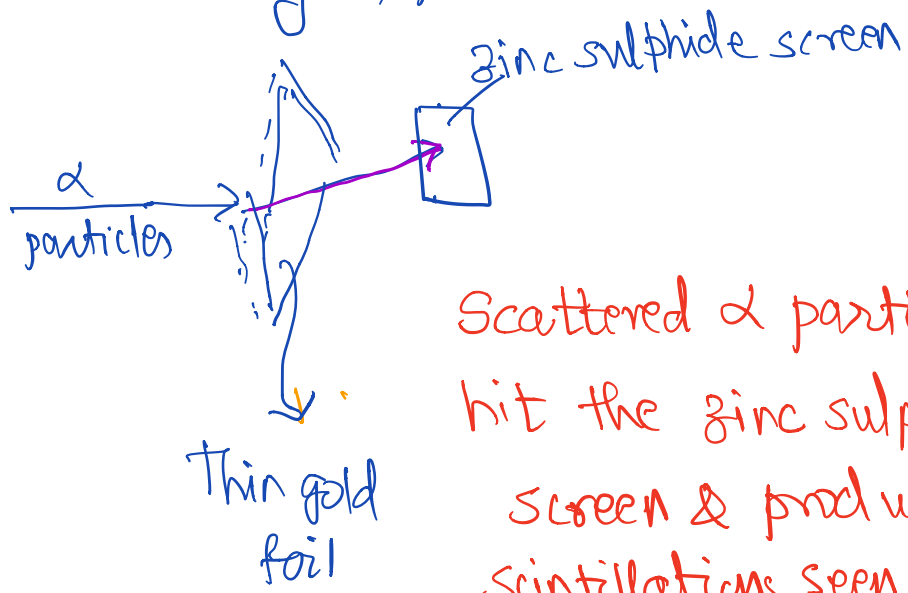
$$\lambda = \frac{h}{p}.$$

The higher the energy of the particle smaller the wavelength and hence smaller is the size of the object that one "see" with these particle beams.

How do we "see" using particles?

Do scattering expts.

After all Rutherford expt. was a scattering expt.



Scattered α particles hit the zinc sulphide screen & produced scintillations seen by a microscope

Observed large angle scatterings

⇒ sometimes α particles went close to the region where charge was concentrated

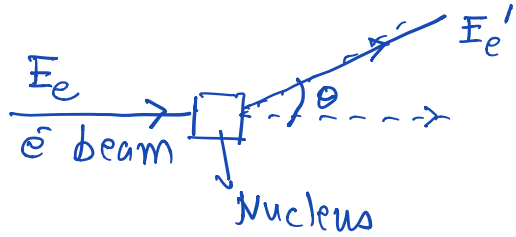
The size was smaller than the distance of closest approach of α ptcle., which will be decided by α particle energy

This is how Rutherford could conclude that the +ve charge in a nucleus is concentrated in a region smaller than 10^{-12} cm.

This then says that if we bombard a nucleus with higher energy ptcles they might be able to penetrate closer and "resolve" a nucleus further

of course with higher & higher energies of projectile we might need to use Quantum Mechanics as we probe smaller & smaller distances

The Hofstadter Expt. was just an extension of Rutherford Expt. in a way



θ : scattering angle

E_e, E_e' : energies of incoming, outgoing e^-

$$e^-(E_e) + A \longrightarrow e^-(E_e') + A$$

The nucleus being very heavy recoils very little. (The electron energies ~ a few 100 MeV)

So it is really potential scattering, $E_e' = E_e$

Question : Is nucleus a point charge or a charge distribution with finite spatial extent.

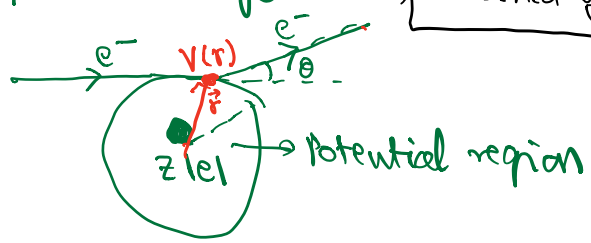
Expt. is simple.

Compare the angular distribution of e^-s scattered thro' an angle θ with that expected for a point charge.

Theory Part : How do we compute expected # of e^-s scattered at an angle θ.

Using a methods of Quantum Mechanics we can do this calculation quite easily.

Let us first do the calculation for a point charge $Z|e|$, Screened by atomic e^-s.

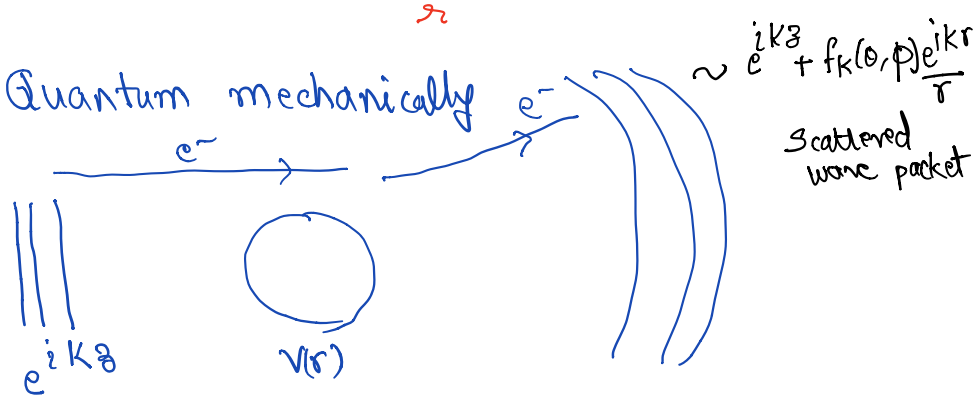


Modifies $V(r)$ to have $e^{-r/a}$ where a: atomic radius

The e^- will come from a potential free region enter the region where it feels the potential.

$$V(r) = \frac{Z|e|e}{r} \times e^{-r/a}$$

Quantum mechanically



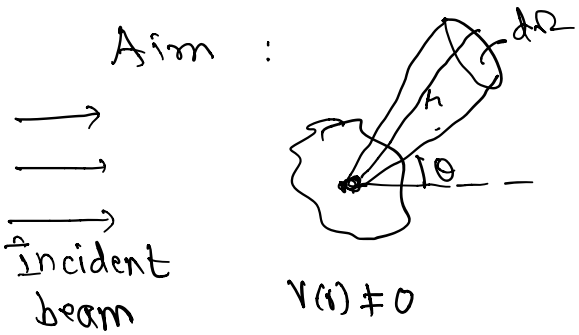
$$|K| = \frac{\sqrt{2mE_e}}{\hbar}$$

free

Effect of potential to be computed using scattering theory

Solve Schrödinger Eq^n. in the region with potential and match the solutions for left & for right. Complicated exercise.

Aim :



$dn \equiv$ # of particles scattered in a solid angle $d\Omega$ around θ, ϕ, per unit time.

$dn \propto F_i \, d\Omega \Rightarrow dn = \sigma(\theta,\phi) \, F_i \, d\Omega.$

F_i ↳ flux

$\sigma(\theta,\phi) =$ differential cross-section

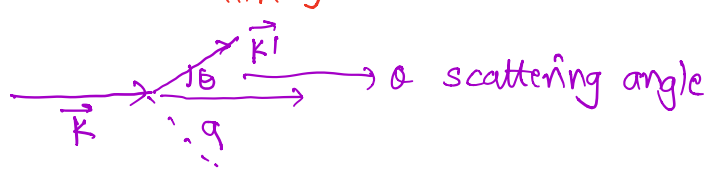
$$\equiv \frac{d\sigma}{d\Omega}$$

Want to calculate $\frac{d\sigma}{d\Omega}$ given $V(\vec{r})$

$$\frac{d\sigma}{d\Omega} = |f_k(\hat{r})|^2 \; ; \quad f_k(\vec{r}) : \text{scattering amplitude.}$$

In an approximation called Born approximation

$$f_k(\vec{k}) = -\frac{m}{2\pi\hbar^2} \int e^{-i\vec{k}'\cdot\vec{r}'} V(r') \, e^{i\vec{k}\cdot\vec{r}'} \, d^3r'$$



$|\vec{k}| = |\vec{k}'|$ (elastic scattering).

For a central potential

$$f_k^{Born}(\theta) = -\frac{m}{2\pi\hbar^2}\int V(r')\, e^{-i(\vec{k}'-\vec{k})\cdot\vec{r}'}\, d^3r'$$

$$\boxed{f_k(\theta) = -\frac{m}{2\pi\hbar^2}\int e^{-i\vec{q}\cdot\vec{r}'}\, V(r')\, d^3r'}$$

This is a bit like form of field amplitude in Fraunhofer diffraction.

Scattering amplitude is F.T. of potential.

$$f_k(\theta) = -\frac{m}{2\pi\hbar^2}\int_0^\infty r'^2 dr'\, V(r')\int_{-1}^{1} d\cos\theta \int_0^{2\pi} e^{-i|\vec{q}|\cdot r'\cos\theta}\, d\phi$$

$$|\vec{q}| = |\vec{k}'-\vec{k}| = q$$

$$= -\frac{m}{2\pi\hbar^2}\int_0^\infty r'^2 dr'\, V(r')\; 2\pi \int_{-1}^{1} d\cos\theta\, e^{-iqr'\cos\theta}$$

$$= -\frac{2m}{\hbar^2}\int_0^\infty V(r')\,\frac{\sin qr'}{qr'}\, r'^2 dr'$$

$$q^2 = |\vec{k}'|^2 + |\vec{k}|^2 - 2|\vec{k}||\vec{k}'|\cos\theta \Rightarrow \boxed{q = 2k\sin\theta/2.}$$

$$q^2 = 4k^2\sin^2\theta/2$$

$\therefore$ For elastic scattering form Screened Coulomb Pot.

$$V(r) = \frac{Z|e|e}{r}\cdot e^{-r/a}$$

a: atomic radius

$\sim$ few Angstroms $\sim 10^{-8}$ cm

$\sim 10^{-10}$ m.

$$f_{Born}(\theta) = +\frac{2m}{\hbar^2}\frac{Ze^2}{}\int_0^\infty \frac{e^{-r'/a}}{r'}\,\frac{\sin qr'}{qr'}\,r'^2\,dr'$$

$$\boxed{f_{Born}(\theta) = \frac{2mZe^2}{\hbar^2}\;\frac{1}{q^2+\frac{1}{a^2}}}$$

q : momentum transfer in units of $\hbar$.

$q \sim |\vec{k}| \sim \sqrt{2mE}/\hbar \Rightarrow \hbar q \sim O(\text{MeV})$

For the energies of e^- of order MeV and above

$$\boxed{\frac{1}{a} << q}$$

Can you argue why?

Remember Binding Energies of an e^- in atom

$\sim$ eV. $\Rightarrow \frac{\hbar}{a} \sim O(\text{eV})$

$$\Rightarrow \boxed{\frac{1}{a} << q}$$

$$f_{Born}(\theta) = \frac{2mZe^2}{\hbar^2}\;\frac{1}{q^2}$$

$\Rightarrow$

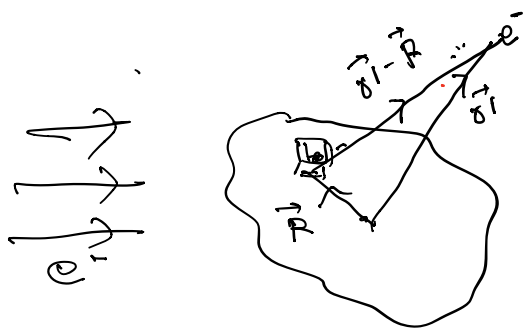
$$\frac{d\sigma}{d\Omega} = \frac{4m^2Z^2e^4}{\hbar^4}\;\frac{1}{16k^4\sin^4\theta/2}$$

$$= \frac{m^2Z^2e^4}{4p^4\sin^4\theta/2} = \frac{Z^2e^4}{16E^2\sin^4\theta/2}$$

(substitute $E = \frac{p^2}{2m}$)

= Rutherford formula.

Now we will see how this will change if the Nucleus is <u>NOT</u> a point charge but a charge distribute $\rho(\vec{R})$ $\ni$ $\int \rho(\vec{R})\, d^3R = 1$



$$\therefore \quad V(\vec{r}') = -Ze^2 \int_{\substack{\text{Volume} \\ \text{of charge distn.}}} \frac{\rho(\vec{R})\, d^3\vec{R}}{|\vec{r}' - \vec{R}|}\, e^{-|\vec{r}' - \vec{R}|/a}$$

Recall

$$\therefore \quad f_{Born} = \frac{-m}{2\pi\hbar^2} \int V(\vec{r}')\, e^{-i\vec{q}\cdot\vec{r}'}\, d^3r'$$

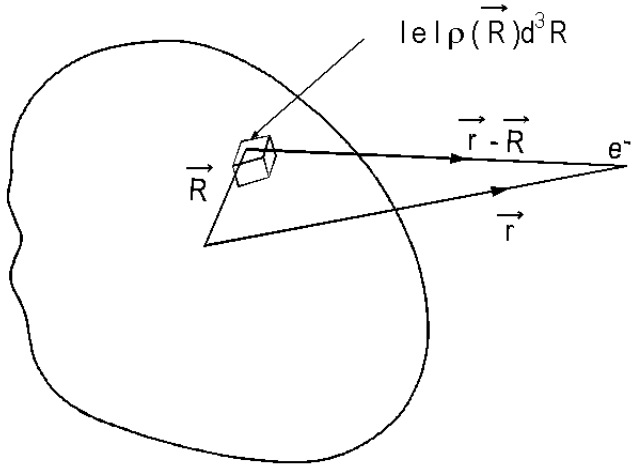
$$= \frac{mZe^2}{2\pi\hbar^2} \int d^3r' \int_V \frac{d^3R\, \rho(\vec{R})}{|\vec{r}' - \vec{R}|}\, e^{-|\vec{r}' - \vec{R}|/a} \times e^{-i\vec{q}\cdot\vec{r}'}$$

$$= \frac{mZe^2}{2\pi\hbar^2} \int_V d^3R\, \rho(\vec{R}) \int \frac{d^3r'}{|\vec{r}' - \vec{R}|}\, e^{-|\vec{r}' - \vec{R}|/a} \times e^{-i\vec{q}\cdot\vec{r}'}$$

$$= \frac{2mZe^2}{\hbar^2} \frac{F(q^2)}{q^2 + 1/a^2}$$

Where $\boxed{F(q^2) = \int e^{i\vec{q}\cdot\vec{R}}\, \rho(\vec{R})\, d^3R}$

Again $\frac{1}{a} << q$



The kinematics of this scattering process is defined in terms of angle θ.

If $\vec{Q} = \vec{p} - \vec{p}'$, normally convenient to use Q^2 instead of θ

If $\rho(\vec{R})$ is the space distribution of the scattering centres one can show that

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{charge distn.}} = |F(Q^2)|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}}$$

where $F(Q^2)$ is the Fourier Transform of the 'normalised' charged distribution.

Thus spatial distribution will modify the Q^2 dependence compared to the expectation for a point and for a point $F(Q^2)$ will be a *constant*.

In fact it can be shown that at $Q^2 \ll 1/ < R^2 >$,

$$F(Q^2) \sim 1 - < R^2 > Q^2$$

This then explains why Rutherford found the nucleus to be pointlike eventhough we NOW know it to have size of the order of a few Fermis.

Our ability to infer and study structure of an object from scattering experiments is possible only when $< R^2 > Q^2 \simeq 1$. I.e. smaller the spatial extension higher the energy required.

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{m^2 z^2 e^4}{4\hbar^4 k^4 \sin^4\theta/2} \, |F(q^2)|^2}$$

Rutherford (point like)

modification due to finite size

$\therefore$ The effect of finite size will change q^2 dependence of $\frac{d\sigma}{d\Omega}$.

For elastic scattering the angular dependence.

Let us check what $F(q^2)$ is for a point charge

$$\rho(\vec{R}) = \delta^3(\vec{R}) \Rightarrow F(q^2) = \int e^{i\vec{q}\cdot\vec{R}} \rho(\vec{R}) \, d^3R$$

$$= \int e^{i\vec{q}\cdot\vec{R}} \delta(\vec{R}) \, d^3R$$

$$= 1.$$

For $\rho(\vec{R}) = \rho(R)$ i.e spherically symmetric

$$F(q^2) = \int e^{i\vec{q}\cdot\vec{R}} \rho(R) R^2 dR \, d\cos\theta \, d\phi$$

$$= 4\pi \int_0^\infty \rho(R) \, dR \, R^2 \frac{\sin qR}{qR}$$

If $qR \ll 1$ where $\rho(R)$ is nonzero

then we can expand $\sin qR / qR$

$$F(q^2) = 4\pi \int dR\, R^2 \rho(R) - \frac{q^2}{6} \int_0^\infty 4\pi R^2 \rho(R)\, dR$$

since $\int \rho(\vec{R})\, d^3R = 1$

$$\boxed{F(q^2) = 1 - \frac{q^2 \langle R^2 \rangle}{6}}$$

$F(q^2)$ starts differing from 1 as qR increases

one can see the effects as $q\langle R \rangle$ becomes large.

As long as $\boxed{q \ll \frac{1}{\langle R \rangle}}$ $F(q^2) \approx 1$

We need values of $q \sim O(\langle R \rangle)^{-1}$

So for nuclei which are $\sim$ fm we needed

e^- beams of energy $\sim$ 100 MeV.

$$\boxed{\hbar q \approx \frac{\hbar}{\langle R \rangle \text{ fm}}}$$

You can check for $\langle R \rangle \approx$ fm one

needs $E \sim$ MeV.

Different forms of $\rho(\vec{R})$ will give diff. $F(q^2)$.

By measuring $F(q^2)$ one can get information about $\rho(\vec{R})$

Eg.

$$\rho(\vec{R}) = \delta^3(\vec{R}) \Rightarrow F(q^2) = 1$$

(we already saw this)

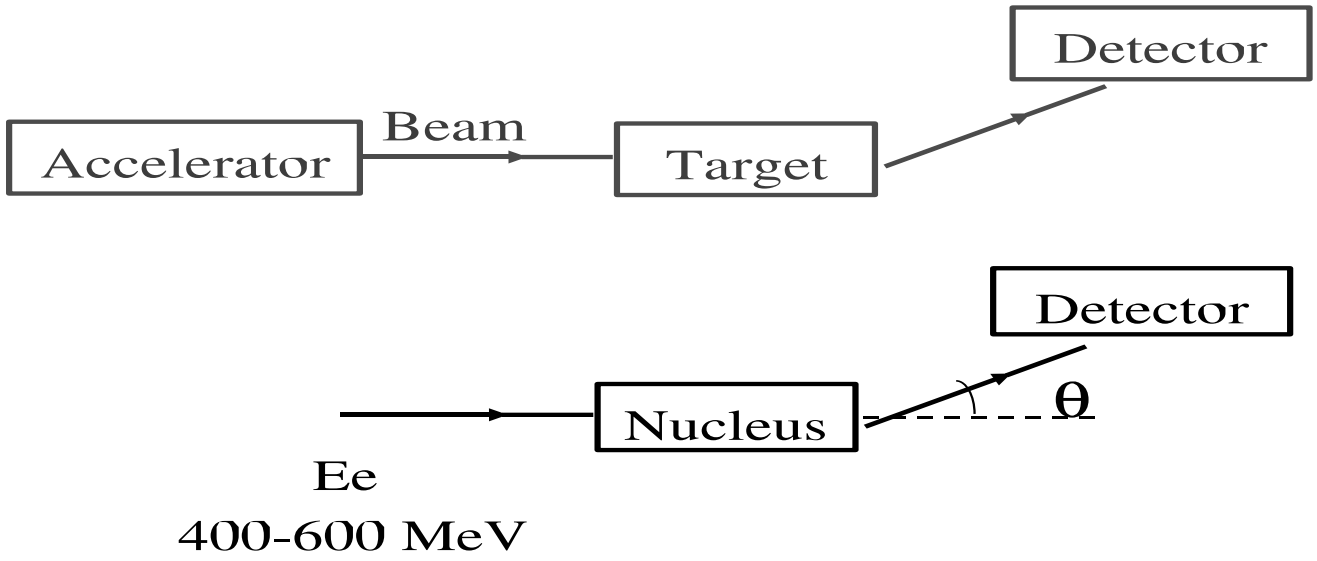
$$\rho(\vec{R}) = \frac{m^2}{4\pi R} e^{-mR} \Rightarrow F(q^2) = \frac{1}{1+q^2/m^2}$$

$$\rho(\vec{R}) = \frac{m^3 e^{-mR}}{8\pi} \Rightarrow F(q^2) = \frac{1}{(1+q^2/m^2)^2}$$

etc.

A bit like reconstructing crystal structure from e^-/n diffraction patterns.

The Hofstadter Experiment: The nucleus/proton version of Rutherford Scattering experiment. **Stanford Linear Accelerator: S.L.A.C.**



Note similarity with Rutherford experiment. The $\lambda_e \sim$ a 1000-10,000 times smaller than λ_α. **Count the number of electrons scattered at an angle θ compare it with the number expected for a 'point' nucleus/proton.**

from : Interactions.org

This is how one measured the "size" of a nucleus and also of a proton.

Further similar methods showed us that the proton is made up of quarks

But that is a story for another day!