Story of prediction of MW and the SM.

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MW prediction and the SM.

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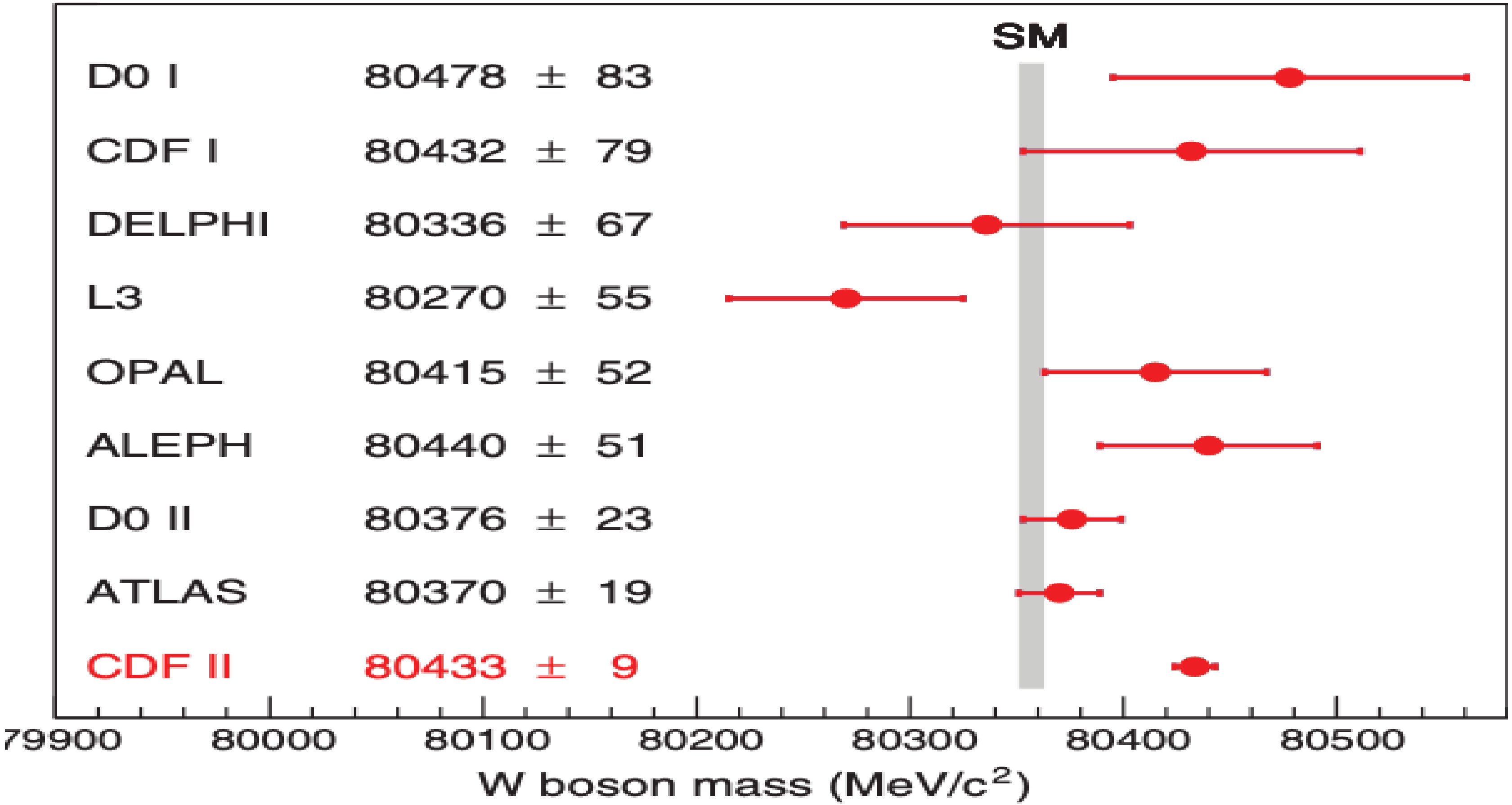
IISER - Mohali.

- MW - Unitarity (1957?)
- MW - prediction -1967.
- MW - prediction and testing the SM - 1984.
- MW - prediction and testing the SM - 1995 .
- MW - prediction and testing the SM - 2012
- MW - prediction and probing the BSM? Are we there yet?


SM
D0 I 80478 ± 83
CDF I 80432 ± 79
DELPHI 80336 ± 67
L3 80270 ± 55
OPAL 80415 ± 52
ALEPH 80440 ± 51
D0 II 80376 ± 23
ATLAS 80370 ± 19
CDF II 80433 ± 9
79900 80000 80100 80200 80300 80400 80500
W boson mass (MeV/c^2)

Existence of W suggested by Schwinger to cure problems of Fermi theory with unitarity.

The first prediction for M_W is from unitarity violation which happens around 300 GeV.

M_W should be bounded by a few hundred GeV.

"Model of Leptons" : 1967 Weinberg Paper.

Number of parameters of the EW sector of the SM:

$SU(2)$ coupling g_2, $U(1)$ couplings g_1, vev v , λ and μ^2.

EW symmetry breaking condition relates v, λ and μ^2.

The parameters are then g_2, g_1, v and λ.

Masses of all the bosons in the theory controlled by these four.

$$M_W = \frac{v}{2} g_2, \quad M_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2} = \frac{M_W}{\cos\theta_W}, M_H = \sqrt{2\lambda}v.$$

where

$$\tan\theta_W = \frac{g_1}{g_2}, \; e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \;, \; \sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$$

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} \Rightarrow v^2 = \frac{1}{\sqrt{2}G_\mu} \Rightarrow v \simeq 250 \text{ GeV}$$

.

We can trade g_2, g_1, v for $G_\mu, \sin\theta_W$ and e.

We get

$$M_W = 2^{-5/4} G_\mu^{-1/2} \frac{e}{\sin\theta_W} \simeq \frac{37}{\sin\theta_W} \text{ GeV}$$

This gives us the prediction for

$$M_W > 37 \quad \text{GeV} \quad , \quad M_Z > M_W$$

At this stage M_W, M_Z depend on $G_\mu, \sin\theta_W$ and e.

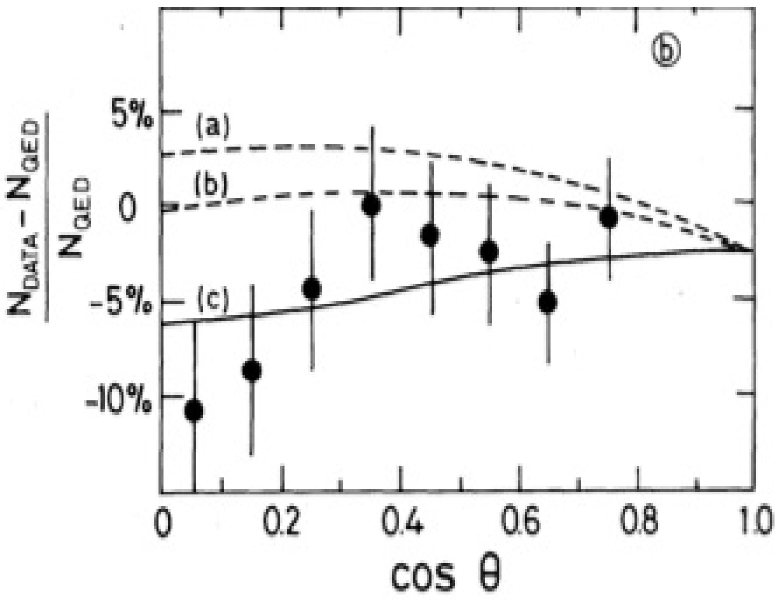
Once we had the measurement of the neutral current processes beginning 1974 and an extraction of $\sin^2\theta_W$ from those measurements we had a prediction for M_W, M_Z *in the framework of the SM*.

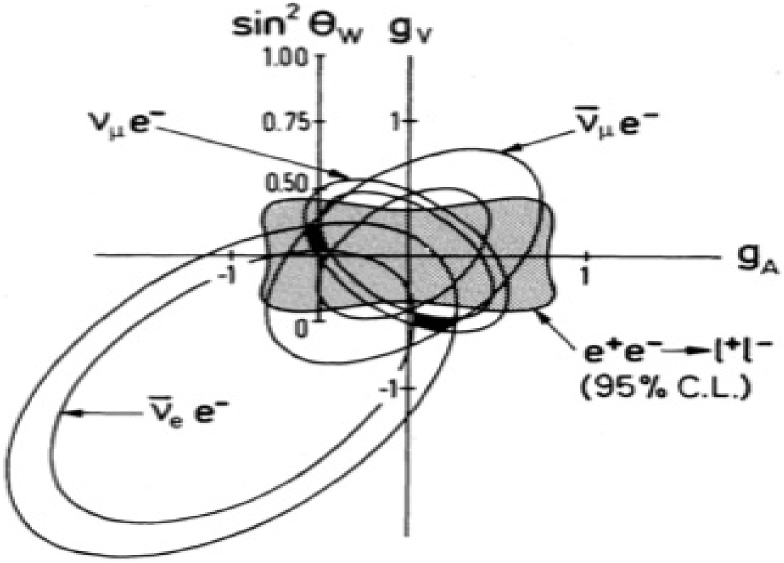
How to determine $\sin\theta_W$?. Couplings g_Z, g_W of W, Z to all fermions predicted in terms of $\sin\theta_W$ and G_μ.

Only the neutral current couplings depend on $\sin^2\theta_W$

$$
\begin{aligned}
g_L^f &= T_3(f_L) - \sin^2\theta_W\ Q_f, \quad g_V^f = T_3(f_L) + T_3(f_R) - 2\ Q_f \sin^2\theta_W \\
g_R^f &= T_3(f_R) - \sin^2\theta_W\ Q_f, \quad g_A^f = T_3(f_L) - T_3(f_R)
\end{aligned}
\tag{1}
$$

Process	σ
$\nu_\mu + e^- \to \mu^- + \nu_e$	$A\ s\ (g_L^\nu)^2 (g_L^e)^2$
$\nu_\mu + e^- \to \nu_\mu + e^-$	$A\ s\ (g_L^\nu)^2[(g_L^e)^2 + \frac{1}{3}(g_R^e)^2]$





$\sin^2\theta_W = 0.27 \pm 0.08$. Validated the EW unification idea.

This gave GSW their Nobel Prize but the measurement of $\sin^2\theta_W$ was poor and precision for predicted M_W, M_Z was ~ 10 GeV.

A better measurement of $\sin_W^\theta$ came from νN scattering experiments, assuming the SM: $\sin^2\theta_W = 0.229 \pm 0.009$ (One assumed doublet Higgs)

This gave a SM prediction (indirect) for the masses :

$M_W \simeq 78.15 \pm 1.5$ GeV; $M_Z \simeq 89 \pm 1.3$ GeV.

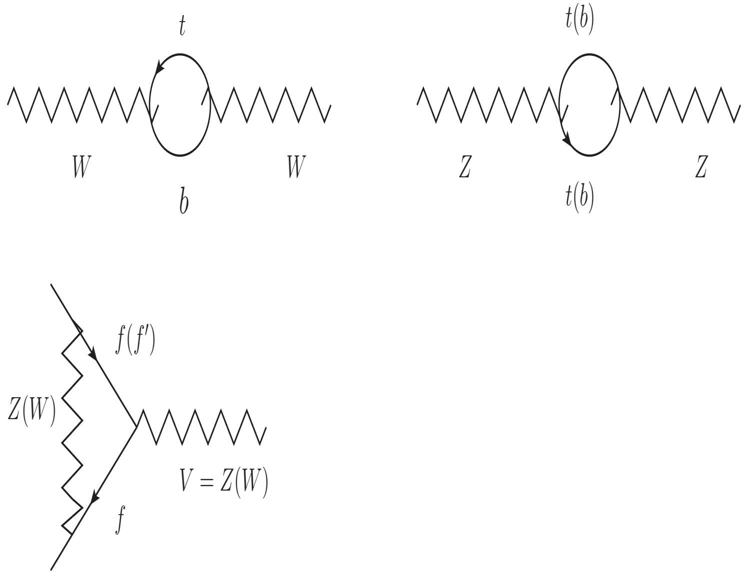
The UA-1/UA-2 measurement was

$M_W = 80 + 10 - 6$ GeV

$M_Z = 91.9 \pm 1.3 \pm 1.4$ GeV.

SM prediction agreed within errors with the **measured** values. (Rubbia and Van der Meer got their Nobel prize for this).

So far one used only tree level relations. The SM is a QFT. There will be loop corrections.

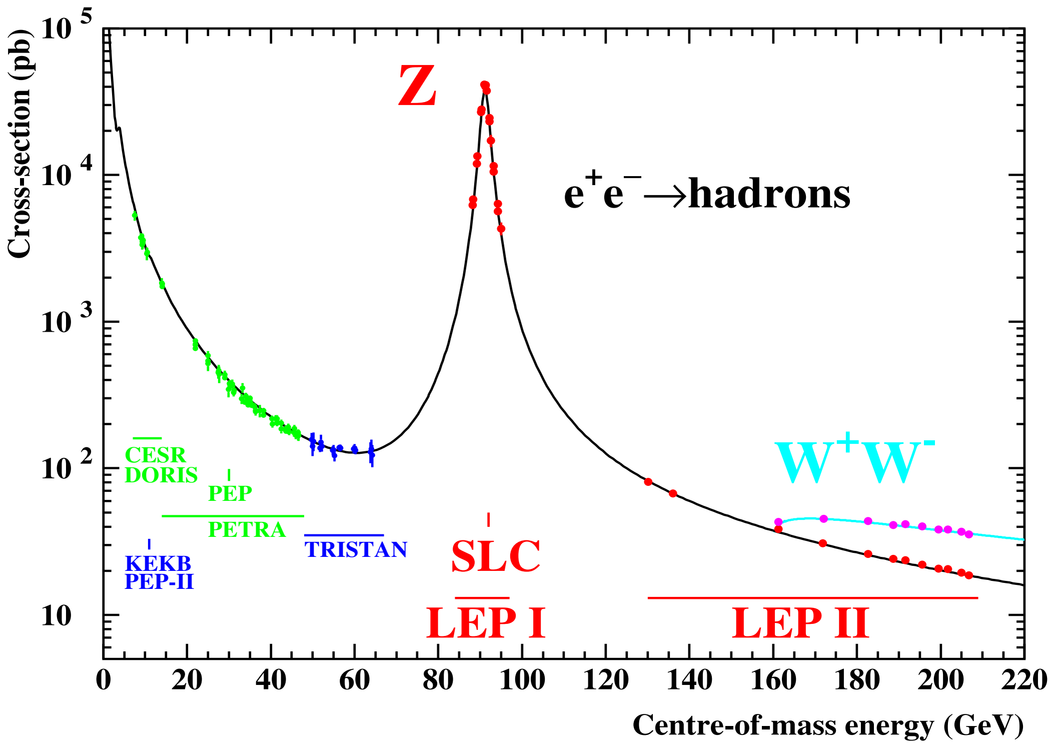


$$\rho_{corr} = 1 + \Delta\rho$$

$$\Delta\rho \simeq \frac{3 G_F M_t^2}{8\pi^2\sqrt{2}} = 0.01$$

There is also a diagram with h in the loop.

The corrections for the Z and W are different. The dominant corrections come from loop containing the heaviest quarks t, b (and sub dominant ones from h) ρ changes from value 1. (Veltman: screening theorem about the h contribution being small) Before top quark was found, its value was indirectly obtained from measuring ρ.

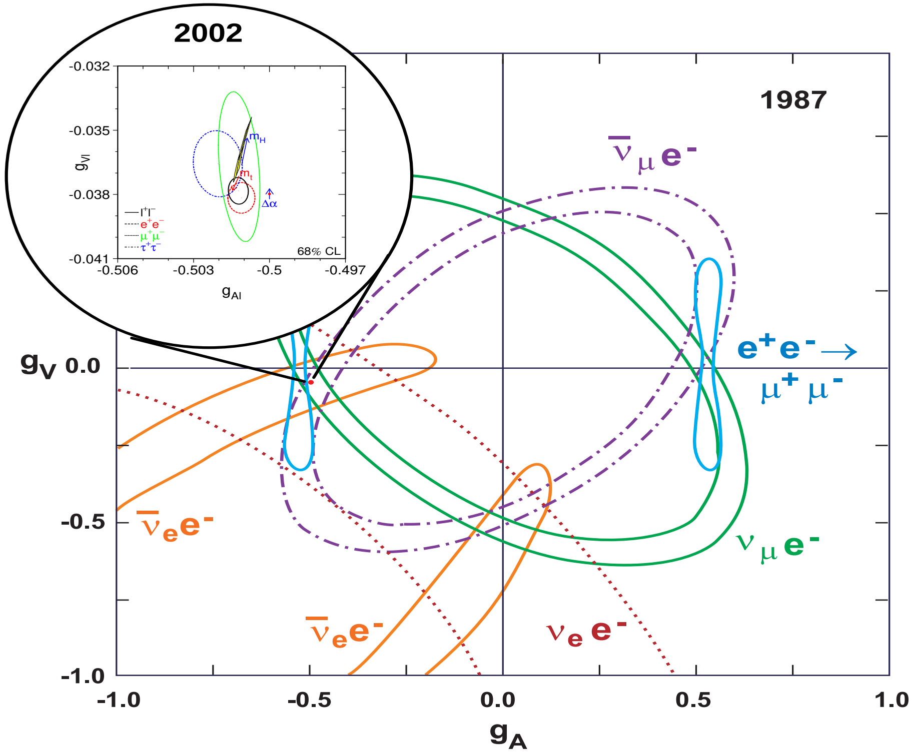


Solid line is the SM fit. Phys. Rept. 427, 257 (2006).

Large electromagnetic and QCD radiative corrections,

Initial state radiation makes the curve asymmetric near the resonance.

These measurements tested the tree level couplings and more!



Enormously more precise measurements.

Agreement with SM prediction would have been impossible unless the predicted values included higher order corrections, calculated in perturbation theory.

Recall correction to $\Delta\rho$ is 1% . The measurement is accurate to 1 part in 100 or better to see confirm this. Large mass of the t made this effect measurable!

Analog of $(g-2)_\mu$ for QED!

Accurate direct measurements of M_W, M_Z were now available. What about SM predictions for these?

Use accurate measurement of M_Z. Trade now $\sin\theta_W$ for M_Z. Given α_{em}, M_Z, G_μ one can calculate M_W using tree level relations.

$\alpha_{em} = 1/137.0359895(61)$, $G_\mu = 1.16637(1) \times 10^{-5} GeV^{-2}; MZ =$ 91.1875 $\pm$ 0.0021 GeV note the precision 2 MeV.

Calculate M_W using the tree level relation

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{\pi\alpha}{2M_W^2(1-M_W^2/M_Z^2)}$$

$M_W^{tree} = 80.939$ GeV and $M_W^{expt} = 80.385 \pm 0.015$ GeV. experimental precision 15 MeV

Loop level calculations required for M_W prediction.

Logical steps in Precision testing of the SM and the indirect limits:

- SM has three parameters g_2, g_1 and v. All the SM couplings, gauge boson masses functions of these.

- A large number of EW observables measured quite accurately.

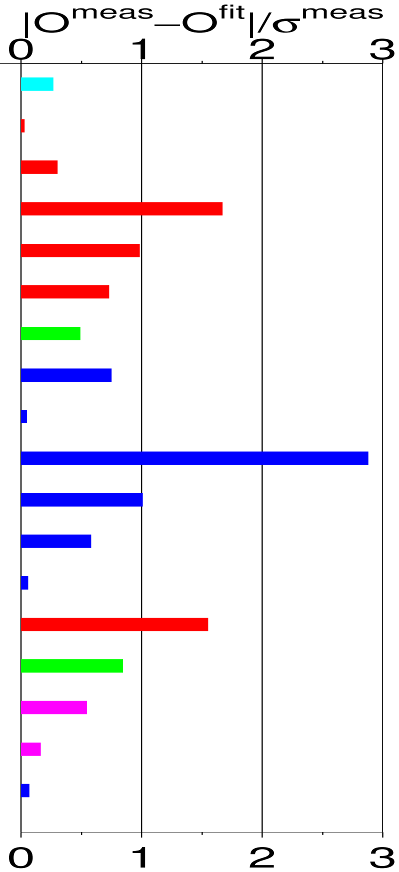
- M_Z, α_{em} and G_F are most accurately measured. Trade g_2, g_1 and v for these.

- All observables depend on these three apart from M_f (mainly M_t) *and* M_h, and of course α_s.

- Calculate all observables using **1 loop EW** radiative corrections which can be computed in a renormalisable quantum field theory.

- Compare with data, make a SM fit. Tests the SM at loop level.

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



March 2011

see http://lepewwg.web.cern.ch

March 2011:

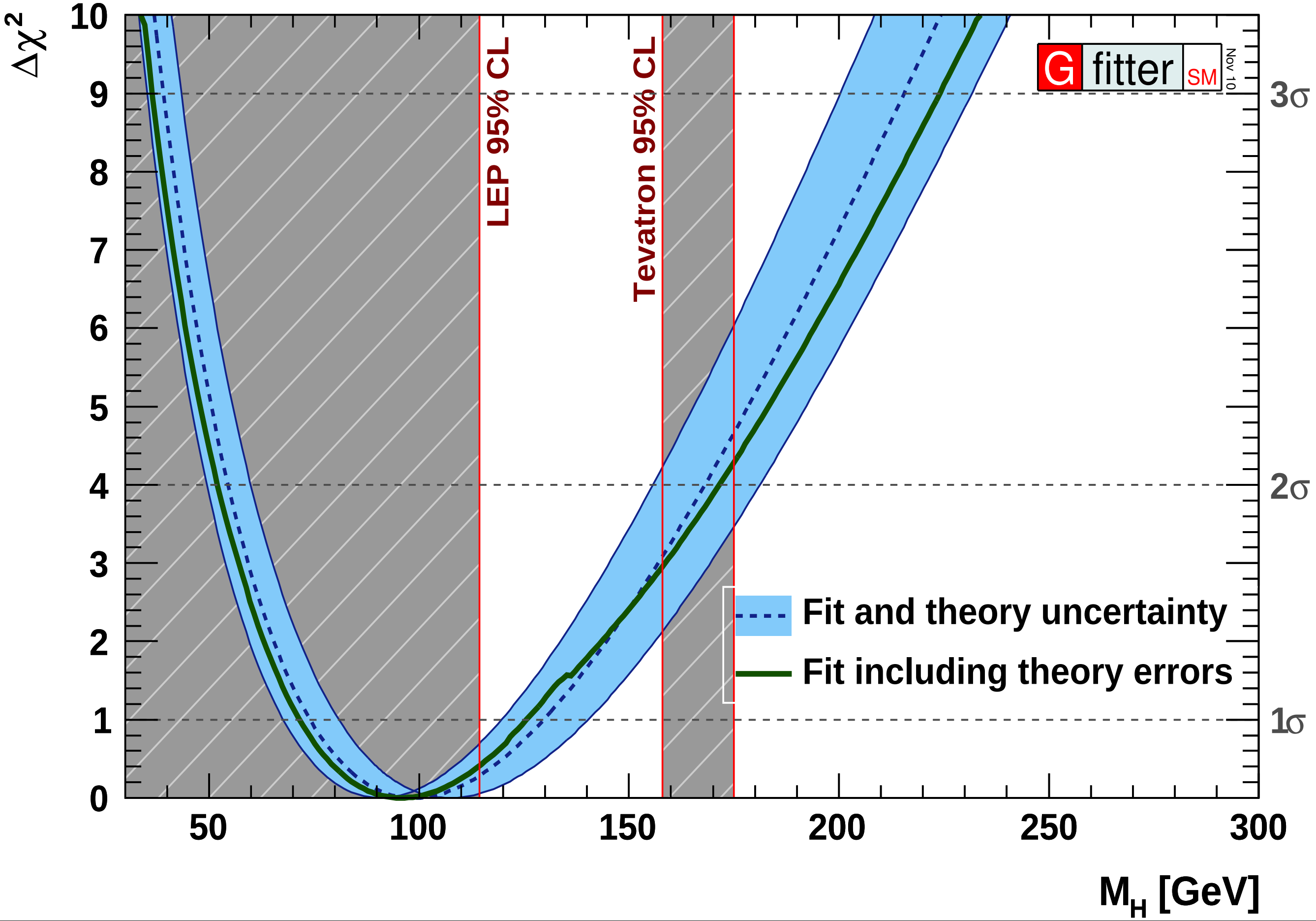
$M_W = 80.385 \pm 0.015$ GeV (direct measured), 80.377 GeV (theory prediction indirect, difference from measured value less than 1 σ)

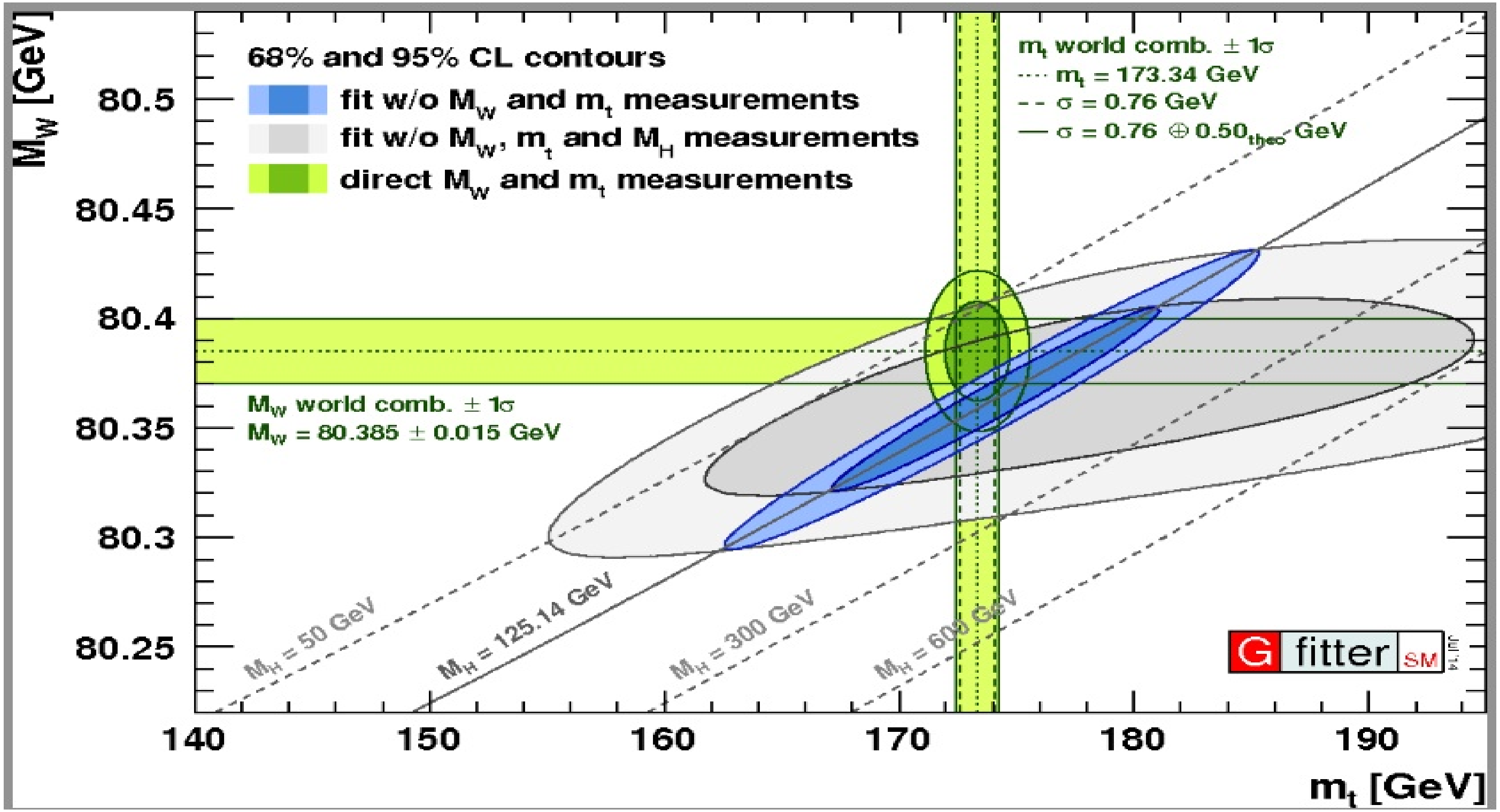
$m_t = 173.20 \pm 0.90$ GeV (measured) 172.26 GeV (theory)

In fact before top mass was measured at the Tevatron the fits made a prediction for it. The agreement between measurement and prediction was a triumph. Veltman and 't Hooft got the Nobel prize only after this happened!

Once top was found and M_t measured the game was to predict M_H.

Now fast forward to 2012: dawn of Higgs discovery. Higgs mass in the SM should be less than 160 GeV (Indirect information!)





SM rocks! *At LOOP level*. M_W slightly larger than the fit prediction.!

Just like the 1987 measurements of g_A–g_V were put under a magnifying glass by LEP-I, LEP-II measurements and we 'predicted' top, higgs masses within the SM!

The hadronic colliders increased the precision of measurement of M_t and M_W. The extraction from precision fits was made more precise by increasing the precision of calculations.

Summary of Numbers from 2013 (Hollick, Weiglein et al :JHEP 12, 2013, 084.):

ATLAS: $M_H = 125.5 \pm 0.2 \pm 0.6$ GeV, CMS : $M_H = 125.7 \pm 0.3 \pm 0.3$ GeV, ATLAS-CMS combinations: 125.64 ± 0.35 GeV

$G_\mu = 1.1663787 \times 10^{-5}, M_Z = 91.1875, \alpha_s(M_Z) = 0.1180, \Delta\alpha_{had} = 0.02757$

$M_W^{fit}(M_t = 173.2, M_H^{SM} = 125.64) = 80.361$ GeV

This *SM prediction* is indeed below the measured world average but by only 1.5σ.

Uncertainties in these values because of errors in our knowledge of M_t, M_Z and $\Delta\alpha_{had}$ is $\sim \mathcal{O}$ 4 MeV. Additional errors due to missing higher orders.

Current top mass measurement: 171.77 ± 0.38 GeV. The one used in plot (world average) I will show you is 172.47 ± 0.46 GeV.

CDF result has higher central value and increased precision! $M_W = 80433 \pm 6.4 \pm 6.9 = 80433.5 \pm 9.4$ MeV

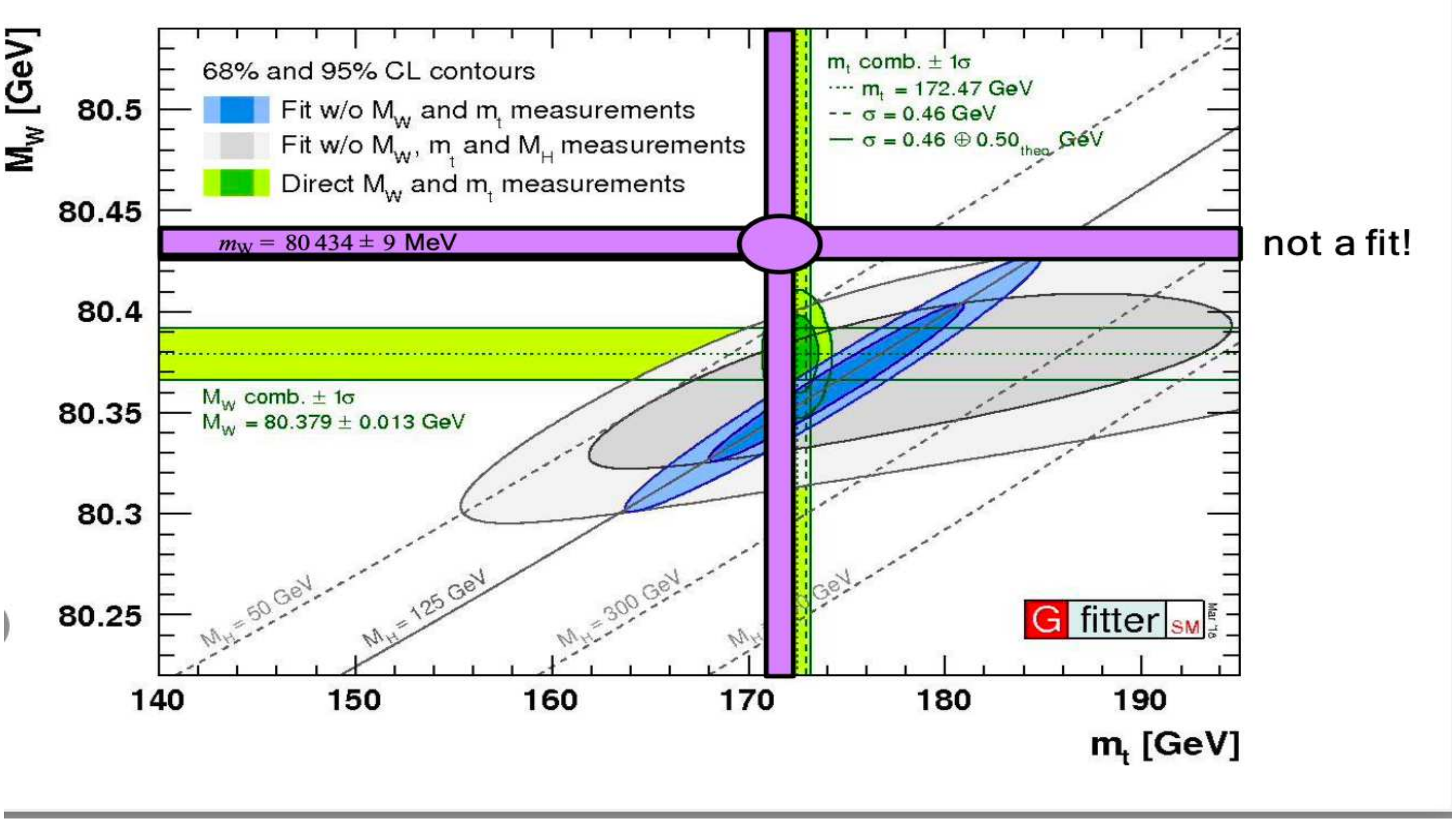
The net precision is 9 MeV. This now starts being competitive with the uncertainties in the 'indirect' theory prediction.

Important to note

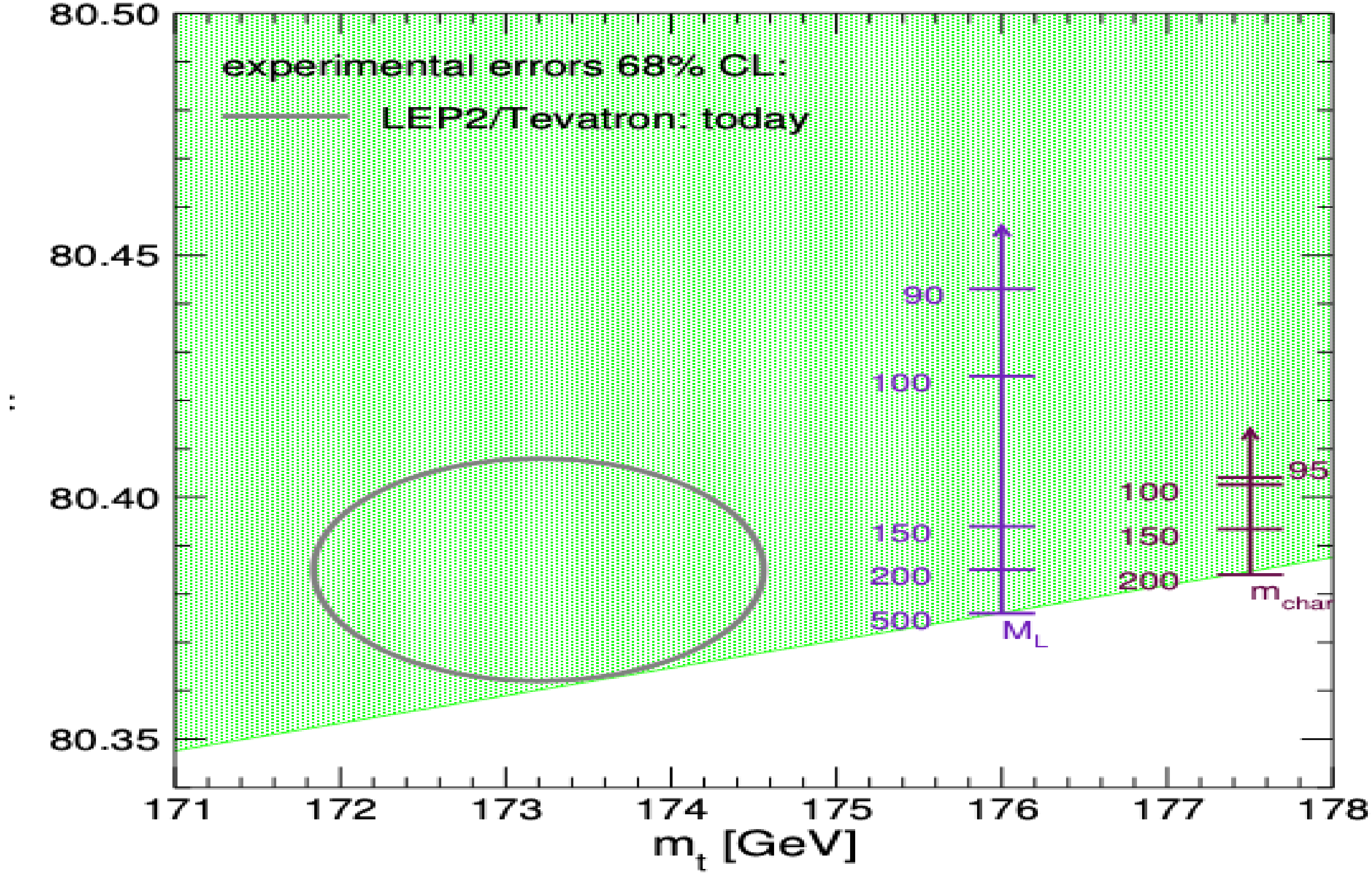
1) The central value has changed by 13.5 MeV from the value obtained by CDF in the analysis of one fourth of the data

2)It is 3σ above all the other hadronic collider measurements.

3)It also differs significantly from the LEP-II value of 80.385 ± 0.015 GeV.
LHCb value (April 2022) $80354 \pm 23_{stat} \pm 10_{exp} \pm 17_{theory} \pm 9_{PDF}$ MeV.



(From Martijn Mulders).



Prediction of M_W in MSSM for light electroweak particles.

Clearly the measurement has potential to indicate BSM physics from this measurement of M_W.

On case where we are actually looking under the lamp.

Important to assess the precision and also consistency with other measurements.